Unit 6
(Polynomial Functions)

Name: _____

Date: _____ Block: _____

Topic 1: Polynomial Operations

6.1

Simplify each expression. Final answers should be written in standard form. 2-4 name by degree and terms.

1. $(-4m^2n)^4 \cdot \frac{1}{6}m^{-10}n^{-4}$

$(-4)^4(m^2)^4 n^4 \cdot \frac{1}{6} \cdot \frac{1}{m^{10}} \cdot \frac{1}{n^4}$

$\frac{256m^8n^4}{6m^{10}n^4}$ → m^{-2} → $\frac{128}{3m^2}$

2. $(8a^2 - 6 - 8a) + (1 - 6a - 7a^2)$

$a^2 - 14a - 5$

quadratic trinomial

$a^2 - 14a - 5$

3. $(6x - 7x^2 + 7) - (5x^2 + 2x - 2x^3 - 1)$

$6x - 7x^2 + 7 - 5x^2 - 2x + 2x^3 + 1$

$2x^3 - 12x^2 + 4x + 8$

cubic 4-term polynomial

$2x^3 - 12x^2 + 4x + 8$

4. $(y + 4)^3 - 2y(y - 1)$

$(y+4)(y+4)(y+4) - 2y^2 + 2y$

$(y^2 + 8y + 16)(y+4) -$

$y^3 + 8y^2 + 16y + 4y^2 + 32y + 64 - 2y^2 + 2y$

cubic 4-term polynomial

$y^3 + 10y^2 + 50y + 64$

5. $(3k - 6)(k^2 - k + 7)$

$3K^3 - 3K^2 + 21K - 6K^2 + 6K - 42$

$3K^3 - 9K^2 + 27K - 42$

6. $\frac{-8c^6d^4 + 56c^4d^2 - 24c^2d}{8c^2d}$

$\frac{-8c^6d^4}{8c^2d} + \frac{56c^4d^2}{8c^2d} - \frac{24c^2d}{8c^2d}$

$-c^4d^3 + 7c^2d - 3$

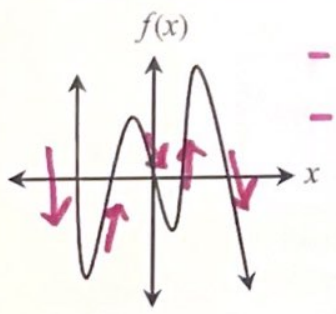
Topic 2: Graphing Polynomial Functions

6.2

2, 4, 6 ... + or −

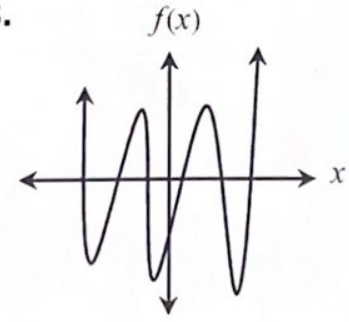
Determine the end behavior, whether the function is an even or odd degree, and the sign of the leading coefficient given each graph below. 3, 5, 7 ...

7.



- odd (5)
- negative

8.



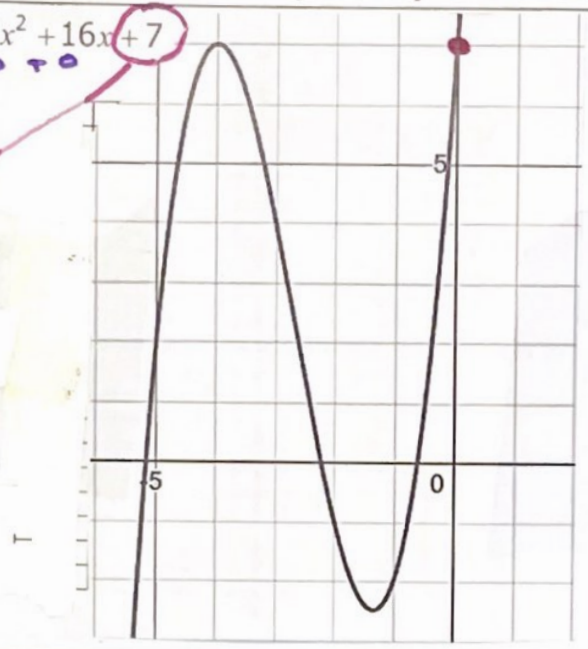
- even (6)
- positive

Graph each function and identify its key characteristics.

9. $f(x) = x^3 + 8x^2 + 16x + 7$

+0 +0

y-int $x=0$ (0,7)



Name: cubic 4 term polynomial

Domain: (-∞, ∞)

Range: (-∞, ∞)

Rel. Maximum(s): (-4, 7)

Rel. Minimum(s): ≈(-1.3, -2.4)

End Behavior: As $x \to \infty$, $f(x) \to$ ∞

As $x \to -\infty$, $f(x) \to$ -∞

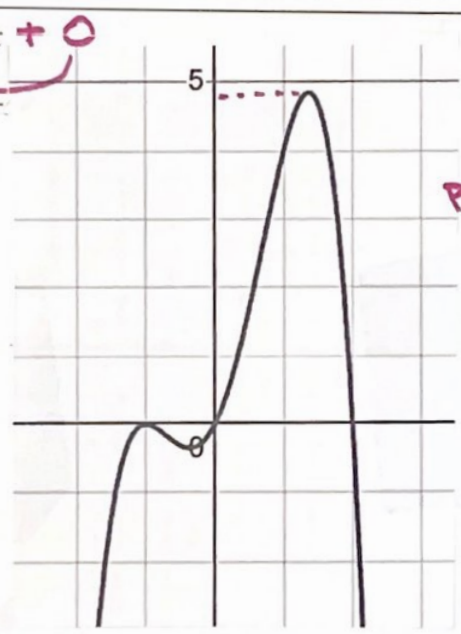
Inc. Intervals: (-∞, -4) U (-1.3, ∞)

Dec. Intervals: (-4, -1.3)

10. $f(x) = -x^4 + 3x^2 + 2x$ + 0

y-int (0,0)

zeros: -1, 0, 2



Name quartic trinomial

Domain: (-∞, ∞)

Range: (-∞, 4.8]

Rel. Maximum(s): (-1, 0); ≈(1.3, 4.8) (point)

Rel. Minimum(s): ≈(-0.4, -0.4) (point)

End Behavior: As $x \to \infty$, $f(x) \to$ -∞

As $x \to -\infty$, $f(x) \to$ -∞

x's make y

Inc. Intervals: (-∞, -1) U (-0.4, 1.3)

Dec. Intervals: (-1, -0.4) U (1.3, ∞)

Identify the zeros (and their multiplicity) of each function below and the effect on the graph.

11. $f(x) = 3x^3(x-1)^2(x-8)$

Zero	Multiplicity	Effect
0	3	crosses x-axis
1	2	touches x axis
8	1	crosses x-axis

12. $f(x) = -x(5x+3)^3(x-2)^8$

Zero	Multiplicity	Effect
0	1	crosses x-axis
-3/5	3	crosses x-axis
2	8	touches x-axis

6.3

13. The graph of a polynomial function has zeros of 0 (multiplicity 2), 1 (multiplicity 2), and 5/2 (multiplicity 2). Write a function in standard form that could represent this function.

$f(x) = x^2(x-1)^2(2x-5)^2$

$= x^2(x^2-2x+1)(4x^2-20x+25)$

$= x^2[4x^4 - 8x^3 + 4x^2 - 20x^3 + 40x^2 - 20x + 25x^2 - 50x + 25)$

$= x^2(4x^4 - 28x^3 + 69x^2 - 70x + 25)$

$f(x) = 4x^6 - 28x^5 + 69x^4 - 7x^3 + 25x^2$

$\frac{5}{2} = x$

$0 = x - \frac{5}{2}$

$0 = 2x - 5$

6.3

Topic 3: Factoring Polynomials steps?

2 Terms	3 Terms	4 Terms
- GCF first - Difference of two squares	- GCF first - $x^4 = (x^2)^2$ subst. - factor quad.	- GCF - grouping

Factoring review

6.4 ↓ 6.8

Factor each polynomial below completely.

14. $9x^3 + 21x^2$

$3x^2(3x+7)$ GCF

$3x^2(3x+7)$

15. $3n^4 - 147$

$3(n^4-49)$ GCF

$3(n^2-7)(n^2+7)$ Diff of 2 sq.

$3(n^2-7)(n^2+7)$

16. $32c^5d - 162cd^3$

$2cd(16c^4 - 81d^2)$ GCF

$2cd(4c^2-9d)(4c^2+9d)$ Diff of 2 sq.

$2cd(4c^2+9d)(4c^2-9d)$

17. $216pq - p^7q$

$pq(216 - p^6)$

↑ not perfect sq

$pq(216-8^6)$

18. $2c^5 - 2c^3 - 60c$

$2c(c^4 - c^2 - 30)$

$(c^2)^2$

$2c(c^2-6)(c^2+5)$

$2c(c^2-6)(c^2+5)$

19. $9y^4 - 7y^2 - 16$

$9(y^2)^2 - 7y^2 - 16$

144, -16, -7, 9

1 144
2 72
3 48
4 36
6 24
8 18
9 16

$(9y^4 - 16y^2) + (9y^2 - 16)$

$y^2(9y^2-16) + 1(9y^2-16)$

$(y^2+1)(9y^2-16)$

diff of 2 sq

$(3y-4)(3y+4)(y^2+1)$

20. $(n^3 + 2n^2) - (36n - 72)$

$n^2(n+2) - 36(n+2)$

$(n^2-36)(n+2)$

diff of 2 sq

$(n-6)(n+6)(n+2)$

21. $(8x^3 - 10x^2) + (28x - 35)$

$2x^2(4x-5) + 7(4x-5)$

$(2x^2+7)(4x-5)$

$(2x^2+7)(4x-5)$

Find all of the zeros of the polynomial and give the complete factorization.

6.5 ↓ 6.8

22. $2x^4 - 48x^2 = 0$

$2x^2(x^2 - 24) = 0$ Does not factor

$2x^2 = 0$ $\quad x^2 - 24 = 0$

$x = 0$ $\quad x^2 = 24$

$x = \pm\sqrt{24}$

$= \pm\sqrt{4 \cdot 6}$

$= \pm 2\sqrt{6}$

$x = \{0, 2\sqrt{6}, -2\sqrt{6}\}$

23. $25x^3 = 64x$

$25x^3 - 64x = 0$

$x(25x^2 - 64) = 0$

$x(5x - 8)(5x + 8) = 0$

$x = 0$ $\quad 5x - 8 = 0$ $\quad 5x + 8 = 0$

$5x = 8$ $\quad 5x = -8$

$x = \frac{8}{5}$ $\quad x = \frac{-8}{5}$

$x = \{0, \frac{8}{5}, \frac{-8}{5}\}$

24. $x^4 + 19x^2 - 20 = 0$

$(x^2)^2 + 19x^2 - 20 = 0$

$(x^2 + 20)(x^2 - 1) = 0$

$(x^2 + 20)(x - 1)(x + 1) = 0$

$x^2 + 20 = 0$

$x^2 = -20$

$x = \pm 2\sqrt{5}\,i$

$x = \{-1, 1, 2\sqrt{5}\,i, -2\sqrt{5}\,i\}$

25. $f(x) = x^4 + 14x^2 - 72$

$f(x) = (x^2)^2 + 14x^2 - 72$

$= (x^2 + 18)(x^2 - 4)$

$= (x^2 + 18)(x - 2)(x + 2)$ zeros

$x^2 + 18 = 0$ $\quad (x - 2) = 0$ $\quad x + 2 = 0$

$x^2 = -18$

$x = \pm 3\sqrt{2}\,i$

$x = \{-2, 2, 3\sqrt{2}\,i, -3\sqrt{2}\,i\}$

26. $f(x) = x^3 - 3x^2 - 13x + 15$ $\quad (x - 1)$ factor

1 | 1 $\quad -3 \quad -13 \quad 15$

$\quad\quad 1 \quad -2 \quad -15$

$1 \quad -2 \quad -15 \quad 0$

$f(x) = (x - 1)(x^2 - 2x - 15)$

$= (x - 1)(x - 5)(x + 3)$

Zero prod. prop

$x = \{1, 5, -3\}$

27. $x^5 = 18x^3 - 81x$

$0 = x^5 + 18x^3 - 81x$

$= x(x^4 - 18x^2 + 81)$

$= x(x^2 - 9)(x^2 - 9)$

$= x(x - 3)(x + 3)(x - 3)(x + 3)$

$0 = x$ $\quad 0 = x - 3$ $\quad 0 = x + 3$

$x = \{0, 3, -3\}$

(x+4) factor

28. $x^4 + 5x^3 + x^2 - 15x = 12$

$x^4 + 5x^3 + x^2 - 15x - 12 = 0$

-1 | 1 5 1 −15 −12
−4 −4 12 12
1 1 −3 −3 0

$f(x) = (x+4)(x^3 + x^2 - 3x - 3)$

$x^2(x+1) - 3(x+1)$

$= (x+4)(x^2-3)(x+1)$

$x^2 = 3$

$x = \pm\sqrt{3}$

$x = \{-1, -4, \sqrt{3}, -\sqrt{3}\}$

29. $2x^3 + 7x^2 - 16x - 56 = 0$

$= x^2(2x+7) - 8(2x+7)$

$= (x^2-8)(2x+7)$

$0 = x^2 - 8$ $\quad 2x + 7 = 0$

$8 = x^2$ $\quad 2x = -7$

$\pm 2\sqrt{2} = x$ $\quad x = \frac{-7}{2}$

$x = \{2\sqrt{2}, -2\sqrt{2}, -\frac{7}{2}\}$

Topic 5: Dividing Polynomials — Long Division

6.6

30. $(12x^2 - 20x + 3) \div (2x - 3)$

$6x - 1$

$2x-3 \overline{)\ 12x^2 + -20x + 3}$

$-(12x^2 - 18x)$

$-2x + 3$

$-(-2x + 3)$

0

$6x - 1$

31. $(y^4 - 7y^3 - 2y + 18) \div (y - 7)$

$y^3 \quad -2$

$y-7 \overline{)\ y^4 - 7y^3 + 0y^2 - 2y + 18}$

$(y^4 - 7y^3)$ ↓ ↓

$0 + 0y^2 - 2y + 18$

$-(-2y + 14)$

4

$y^3 - 2 + \frac{4}{y-7}$