# Unit 4 Test Study Guide
(Exponential & Logarithmic Functions)

Name: _____

Date: _____ Block: _____

Topic 1: Graphing Exponential & Logarithmic Functions

**Describe as an exponential growth or decay.** 5.1

1. $f(x) = 5\left(\frac{2}{3}\right)^x$ Exponential Decay

2. $f(x) = \frac{1}{3}\left(\frac{6}{5}\right)^x$ Exponential Growth

**Graph each function and identify its key characteristics.** 5.1

3. $f(x) = 3^{x+1} - 6$

x intercept $y=0$

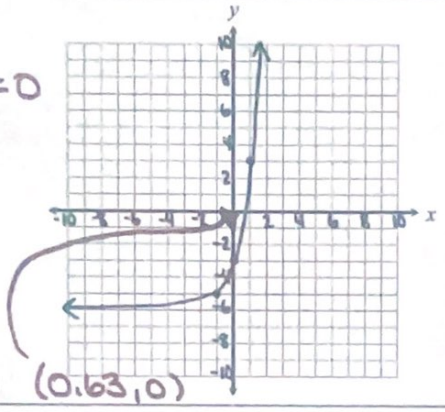
$0 = 3^{x+1} - 6$

$6 = 3^{x+1}$

$\log 6 = \log 3^{x+1}$

$x+1 = \frac{\log 6}{\log 3}$

$x = \frac{\log 6}{\log 3} - 1 \approx 0.63$ (0.63, 0)



Domain: $(-\infty, \infty)$

Range: $(-6, \infty)$

End Behavior:

As $x \to \infty$, $f(x) \to$ $\infty$

As $x \to -\infty$, $f(x) \to$ $-6$

$y$-intercept: $(0, -3)$

Asymptote: $y = -6$

4. $f(x) = \left(\frac{1}{2}\right)^{x-5} + 2$

$y$-intercept:

$f(0) = \left(\frac{1}{2}\right)^{0-5} + 2$

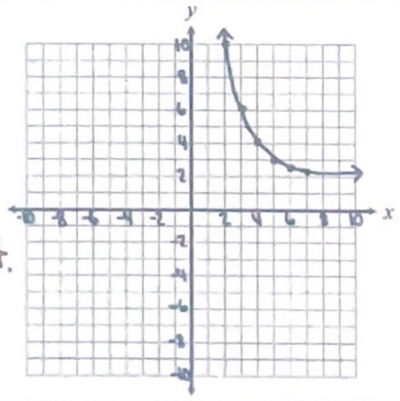
$= \left(\frac{1}{2}\right)^{-5} + 2$

$= 32 + 2$

$= 34$

$(0, 34)$

no x-int.



Domain: $(-\infty, \infty)$

Range: $(2, \infty)$

End Behavior:

As $x \to \infty$, $f(x) \to$ $2$

As $x \to -\infty$, $f(x) \to$ $\infty$

$y$-intercept: $(0, 34)$

Asymptote: $y = 2$

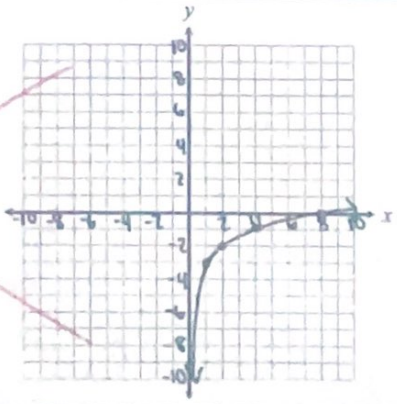
5. $f(x) = \log_2 x - 3$

$y = \log_2(x) - 3$

$+3 \quad +3$

$y + 3 = \log_2(x)$

$2^{y+3} = x$



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

End Behavior:

As $x \to$ $\infty$, $f(x) \to \infty$

As $x \to$ $0$, $f(x) \to -\infty$

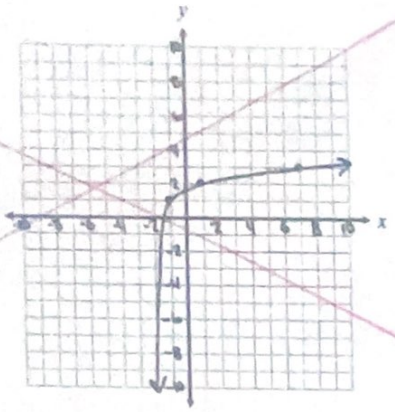
$x$-intercept: $(8, 0)$

Asymptote: $x = 0$

6. $f(x) = \log_3(x+2)+1$

$y = \log_3(x+2)+1$
$-1 \quad\quad -1$
$y-1 = \log_3(x+2)$
$3^{y-1} = x+2$
$-2 \quad\quad -2$
$3^{y-1} - 2 = x$

x-intercept:
$3^{0-1} - 2 = x$
$\frac{1}{3} - \frac{6}{3} = x$
$-\frac{5}{3} = x$



Domain: $(-2, \infty)$

Range: $(-\infty, \infty)$

End Behavior:

As $x \to \infty$, $f(x) \to \infty$

As $x \to -2$, $f(x) \to -\infty$

$x$-intercept: $(-\frac{5}{3}, 0)$

Asymptote: $x = -2$

Topic 2: Exponential vs. Logarithmic Form

| **Write in logarithmic form.** 5.3 | | | |
|---|---|---|---|
| **7.** $8^2 = 64$ <br> $\log_8(64) = 2$ | **8.** $2^{x-4} = 32$ <br> $\log_2(32) = x-4$ | **9.** $10^{2x} = 54$ <br> $\log(54) = 2x$ | **10.** $e^6 = x-2$ <br> $\ln(x-2) = 6$ |
| **Write in exponential form.** 5.3 | | | |
| **11.** $\log_3 27 = 3$ <br> $3^3 = 27$ | **12.** $\log_x 7 = \frac{1}{2}$ <br> $x^{1/2} = 7$ | **13.** $\log_4 90 = x$ <br> $4^x = 90$ | **14.** $\ln x = 38$ <br> $e^{38} = x$ |

Topic 3: Evaluating Logarithms 5.3

| **Evaluate each logarithm. Use the Change of Base Formula when necessary:** $\log_b a = \frac{\log(a)}{\log(b)}$ | | | |
|---|---|---|---|
| **15.** $\log_9 81$ <br> $\log_9 9^2$ <br> 2 | **16.** $\log_{81} 3$ <br> $\log_{81} \sqrt{81} \to \log_{81} 81^{\frac{1}{4}}$ <br> $\frac{1}{4}$ | **17.** $\log_5 \frac{1}{25}$ <br> $\log_5 5^{-2}$ <br> $-2$ | **18.** $\log_6 1$ <br> $\log_6 6^0$ <br> 0 |
| **19.** $\log 63 \approx 1.799$ <br> use common log <br> already base 10 → use calculator | **20.** $\log_7 95 = \frac{\log(95)}{\log(7)}$ <br> $\approx 2.340$ | **21.** $\log_2 78 = \frac{\log(78)}{\log(2)}$ <br> $\approx 6.285$ | **22.** $\ln 42 \approx 3.738$ |

## Topic 4: Properties of Logarithms 5.4

| Product Rule | Quotient Rule | Power Rule |
|---|---|---|
| $\log_b(m \cdot n) =$ $\log_b(m) + \log_b(n)$ | $\log_b\left(\frac{m}{n}\right) =$ $\log_b(m) - \log_b(n)$ | $\log_b m^n =$ $n\log_b(m)$ |

**Condense each expression into a single logarithm.**

**23.** $3 \cdot \log 2 + \log(x-4)$
$= \log(2^3) + \log(x-4)$
$= \log(8) + \log(x-4)$
$= \log(8(x-4))$
$= \log(8x-32)$

**24.** $\frac{1}{2} \cdot \log_5 324 - \log_5 2$
$= \log_5(324^{1/2}) - \log_5(2)$
$= \log_5(18) - \log_5(2)$
$= \log_5\left(\frac{18}{2}\right)$
$= \log_5(9)$

**25.** $3 \cdot \ln 6 - \frac{3}{2} \cdot \ln 4$
$= \ln(6^3) - \ln(4^{3/2})$
$= \ln(216) - \ln(8)$
$= \ln\left(\frac{216}{8}\right)$
$= \ln(27)$

**Expand each expression.**

**26.** $\log_3(x^2y^5)^3$
$= 3\log_3(x^2y^5)$
$= 3(\log_3(x^2) + \log_3(y^5))$
$= 3(2\log_3(x) + 5\log_3(y))$
$= 6\log_3(x) + 15\log_3(y)$

**27.** $\ln\left(\frac{2}{a^3}\right)^4 = 4\ln\left(\frac{2}{a^3}\right)$
$= 4(\ln(2) - \ln(a^3))$
$= 4(\ln(2) - 3\ln(a))$
$= 4\ln(2) - 12\ln(a)$

**28.** $\log_4\sqrt{p^3q^{10}} = \log_4(p^3q^{10})^{1/2}$
$= \frac{1}{2}\log_4(p^3q^{10})$
$= \frac{1}{2}(\log_4(p^3) + \log_4(q^{10}))$
$= \frac{1}{2}(3\log_4(p) + 10\log_4(q))$
$= \frac{3}{2}\log_4(p) + 5\log_4(q)$

## Topic 5: Solving Logarithmic Equations 5.5 & 5.7

**29.** $\log_4(5x+7) = \log_4(2x+31)$

$5x+7 = 2x+31$
$-2x \quad -2x$
$3x+7 = 31$
$-7 \quad -7$
$\frac{3x}{3} = \frac{24}{3}$
$\boxed{x = 8}$

**30.** $\ln(8-13p) = \ln(6p+27)$

$8-13p = 6p+27$
$-8 \quad -8$
$-13p = 6p+19$
$-6p \quad -6p$
$\frac{-19p}{-19} = \frac{19}{-19}$
$\boxed{p = -1}$

**31.** $\frac{1}{2} \cdot \log_8 36 + \log_8(3k+7) = \log_8 132$

$\log_8(36^{1/2}) + \log_8(3k+7) = \log_8(132)$
$\log_8(6) + \log_8(3k+7) = \log_8(132)$
$\log_8(6(3k+7)) = \log_8(132)$
$18k+42 = 132$
$-42 \quad -42$
$\frac{18k}{18} = \frac{90}{18}$
$\boxed{k = 5}$

**32.** $2 \cdot \log(y+5) = \log 20 - \log 5$

$\log(y+5)^2 = \log\left(\frac{20}{5}\right)$
$\log(y+5)^2 = \log(4) \rightarrow (y+5)^2 = 4$
$\sqrt{(y+5)^2} = \sqrt{4}$
$y+5 = \pm 2$

$y+5 = 2$ : $-5 \quad -5$, $\boxed{y = -3}$

$y+5 = -2$ : $-5 \quad -5$, $y = -7$ extraneous

**33.** $\log_2(9m+2)=7$

$2^7 = 9m+2$

$128 = 9m+2$

$-2 \quad -2$

$\frac{126}{9} = \frac{9m}{9}$

$\boxed{m=14}$

**34.** $\frac{5\cdot\ln(2a-1)}{5} = \frac{15}{5}$

$\ln(2a-1)=3$

$e^3 = 2a-1$

$+1 \quad +1$

$\frac{e^3+1}{2} = \frac{2a}{2}$

$a = \frac{e^3+1}{2}$

$\boxed{a \approx 10.543}$

Topic 6: Solving Exponential Equations 5.2 + 5.6 + 5.7

**35.** $64^{x+7} = 4^{5x-3}$

$4^{3(x+7)} = 4^{5x-3}$

$3x+21 = 5x-3$

$-3x \quad -3x$

$21 = 2x-3$

$+3 \quad +3$

$\frac{24}{2} = \frac{2x}{2}$

$\boxed{x=12}$

**36.** $9^{w-8} = \left(\frac{1}{27}\right)^{2w}$

$3^{2(w-8)} = 3^{-3(2w)}$

$2w-16 = -6w$

$-2w \quad -2w$

$\frac{-16}{-8} = \frac{-8w}{-8}$

$\boxed{w=2}$

**37.** $8^{n-5} = 48$

$\log(8^{n-5}) = \log(48)$

$(n-5)\log(8) = \log(48)$

$n-5 = \frac{\log(48)}{\log(8)} + 5$

$+5$

$n = \frac{\log(48)}{\log(8)} + 5$

$\boxed{n \approx 6.862}$

**38.** $2\cdot 3^{4y} - 11 = 61$

$+11 \quad +11$

$\frac{2\cdot 3^{4y}}{2} = \frac{72}{2}$

$3^{4y} = 36$

$\log(3^{4y}) = \log(36)$

$\frac{4y\log(3)}{4\log(3)} = \frac{\log(36)}{4\log(3)}$

$y = \frac{\log(36)}{4\log(3)}$

$\boxed{y \approx 0.815}$

**39.** $e^{a+1} = 65$

$\ln(e^{a+1}) = \ln(65)$

$a+1 = \ln(65)$

$-1 \quad -1$

$a = \ln(65) - 1$

$\boxed{a \approx 3.174}$

**40.** $-3\cdot e^{2m-5} - 7 = -34$

$+7 \quad +7$

$\frac{-3\cdot e^{2m-5}}{-3} = \frac{-27}{-3}$

$e^{2m-5} = 9$

$\ln(e^{2m-5}) = \ln(9)$

$2m-5 = \ln(9)$

$+5 \quad +5$

$\frac{2m}{2} = \frac{\ln(9)+5}{2}$

$m = \frac{\ln(9)+5}{2}$

$\boxed{m \approx 3.599}$