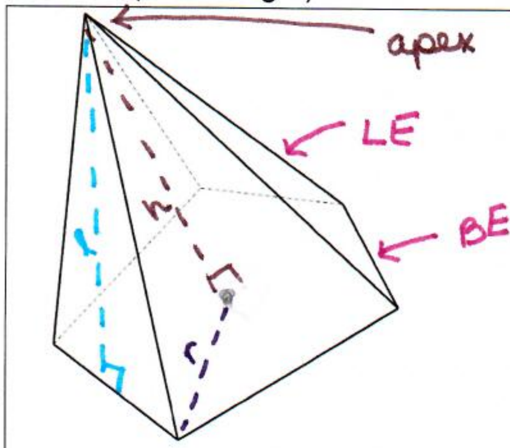
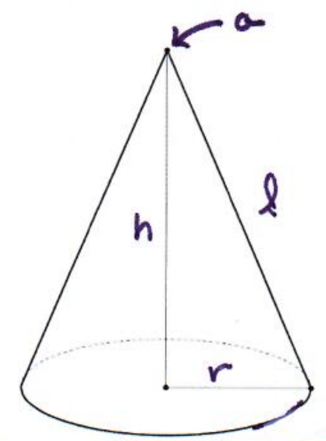
Solid Review Problems

Name ______________________ Date __________ Period __________

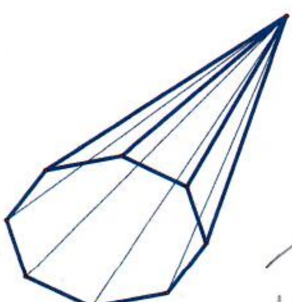
1. For the figures below, lightly shade all bases, **sketch** into the figures and label all that apply:

- h (height)
- a (apex)
- ℓ (slant height)
- LE (Lateral Edge)
- BE (Base Edge)
- r (radius)





2. Find the number of faces edges and vertices.



Bases: 1

Faces: 9

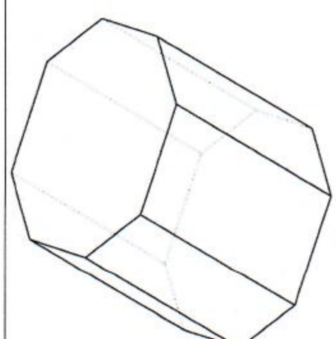
(faces include bases)

Vertices: 9

includes apex

Edges: 16

lateral edges + base edges



Bases: 2

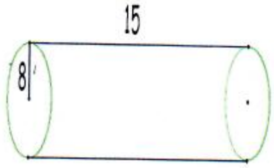
Faces: 10

Vertices: 16

Base Edges: 16

Lateral Edges: 8

3. Find the volume and total surface area of each figure. Write your answer exact and rounded to the hundredth.



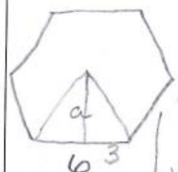
2 bases

$SA = 2\cdot\pi r^2 + 2\pi rh$
$= 2\pi(8)^2 + 2\pi(8)15$
$= 128\pi + 240\pi$
$= 368\pi u^2$
$\approx 1156.11 u^2$

$V = Bh$
$= \pi r^2 h$
$= \pi 8^2 \cdot 15$
$= 960\pi u^3$
$\approx 3015.93 u^3$

A hexagonal prism 5 yd tall with a regular base measuring 6 yd on each edge.

Base



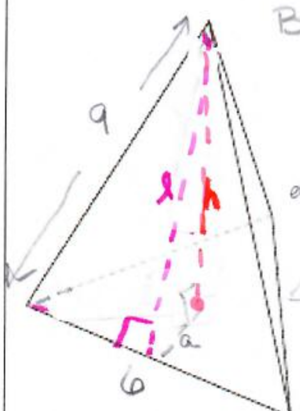
$a = 3\sqrt{3}$
$B = (\frac{1}{2}\cdot 6\cdot 3\sqrt{3})6 = 54\sqrt{3}$
$V = Bh = 54\pi(5) = 270\sqrt{3} u^3$
$V \approx 467.65 u^3$

$SA = 2(54\sqrt{3}) + 6(6\times 5)$
sides faces

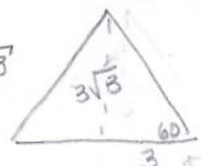
$SA = 108\sqrt{3} + 180 \ u^2$ excact
$SA \approx 491.77 u^2$ rounded

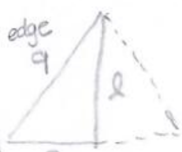
4. Find the volume and surface area of each pyramid. Round your answer to the hundredth.

The base edges are 6 ft, and the lateral edges are 9 ft.



$V = \frac{1}{3}Bh$
$B = \frac{1}{2}6\cdot 3\sqrt{3}$
$= 9\sqrt{3}$
≈ 15.59

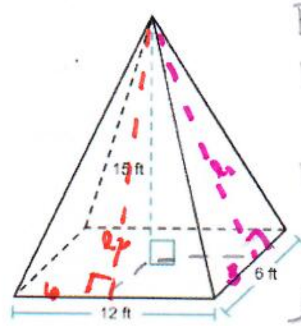




$3^2 + l^2 = 9^2$
$l^2 = 72$

l pyth
$l \approx 8.49$

continued



$B = 12\cdot 6 = 72$
$V = \frac{1}{3}Bh$
$= \frac{1}{3}(72)15$
$V = 360 \text{ ft}^3$

$l_2 = 15^2 + 3^2$
$= 234$
$l_2 = 15.30$

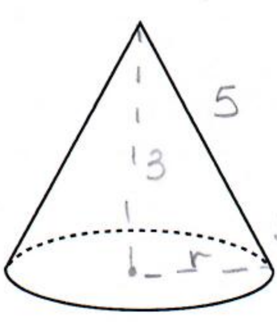
$l_1^2 = 15^2 + 6^2$
$= 261$
$l_1 = 16.16$

$SA = 72 + 2(\frac{1}{2}\cdot 6\cdot 15.30) + 2(\frac{1}{2}\cdot 3\cdot 16.16)$
$\approx$

5. Find the Volume and Total Surface Area of each cone. Leave your answer exact and rounded to the nearest hundredth. Label the slant height, cone height, and radius in each figure.

The slant height is 5 in and the height is 3 in.

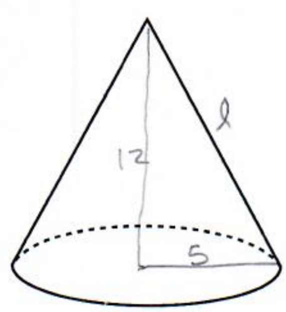
$r = 4$ Pyth Triple



$V = \frac{1}{3}\pi(4)^2(3)$
$= 16\pi \text{ in}^3$
$\approx 50.27 \text{ in}^3$

$SA = \pi r^2 + \pi r l$
$= \pi(4)^2 + \pi(4)5$
$= 16\pi + 20\pi$
$= 36\pi \text{ in}^2$
$\approx 113.10 \text{ in}^2$

The radius is 5 cm and the height is 12 cm.



$V = \frac{1}{3}Bh$
$= \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(5)^2 12$
$= 4\pi 25$
$= 100\pi \text{ cm}^3$
$\approx 314.16 \text{ cm}^3$

$l^2 = 12^2 + 5^2$
$l = 13$
triple

$SA = \pi r^2 + \pi r l$
$= \pi(5)^2 + \pi 5\cdot 13$
$= 90\pi \text{ cm}^2$
$\approx$

4a continue

$V = \frac{1}{3}Bh$

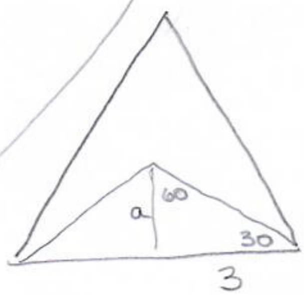
$V = \frac{1}{3}(15.59)(8.31)$

$\approx 43.19 \text{ ft}^3$

$SA = 15.59 + 3(\frac{1}{2} \cdot 6 \cdot 8.49)$

$\approx 15.59 + 76.41$

$\approx 92 \text{ ft}^2$



shorter leg $\quad$ longer leg

$a\sqrt{3} = 3$

$a = \frac{3}{\sqrt{3}} \approx 1.73$

$l^2 = a^2 + h^2$ see diagram

$(8.49)^2 = \left(\frac{3}{\sqrt{3}}\right)^2 + h^2$

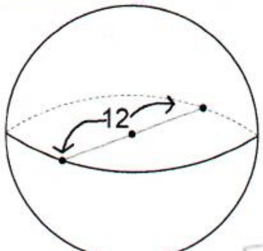
$= \frac{9}{3} + h^2$

$72.08 = 3 + h^2$

$8.31 = h$

6. Leave your answer exact and rounded to the hundredth.

Find the volume and surface area. The diameter is 12 m.



$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3$

$\boxed{V = 288\pi\ m^3 \quad V \approx 904.78\ m^3}$

$SA = 4\pi r^2$

$\boxed{SA = 4\pi \cdot 6^2 = 144\pi\ m^2 \quad \approx 452.39\ m^2}$

The volume of a hemisphere is $54\pi\ ft^3$. What is the surface area?

$V = \frac{1}{2} \cdot \frac{4}{3}\pi r^3$

$54\cancel{\pi} = \frac{2}{3}\cancel{\pi} r^3$

$\overset{27}{\cancel{54}} \cdot \frac{3}{\cancel{2}} = r^3$

$81 = r^3$

$r = \sqrt[3]{81}$

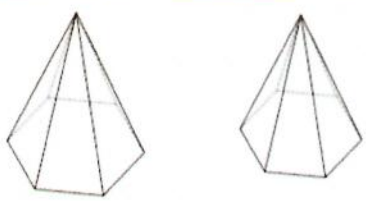
≈ 4.33

$SA = 4\pi r^2$

$= 4\pi(4.33)^2$

$\boxed{SA \approx 235.61\ ft^2}$

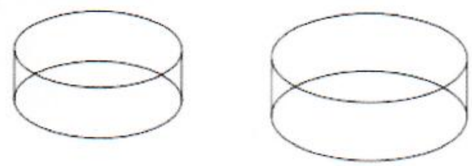
7. The following are similar figures. Find the similarity ratio.



V = 3240 in³ V = 120 in³

$r^3 = \frac{V_1}{V_2} = \frac{324\cancel{0}}{12\cancel{0}} = 27$

$r = \sqrt[3]{27} = 3$ $\boxed{3:1}$

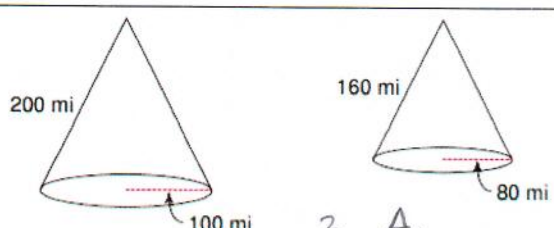


SA = 7π in² SA = 175π in²

$r^2 = \frac{A_1}{A_2} = \frac{7\pi}{175\pi} = \frac{1}{25}$

$r = \sqrt{\frac{1}{25}} = \frac{1}{5}$ $\boxed{1:5}$

8. Answer the following regarding similar figures. Do not calculate the areas or volumes to determine.



$r = \frac{20\cancel{0}}{16\cancel{0}} = \frac{5}{4}$

$r^2 = \frac{A_1}{A_2}$

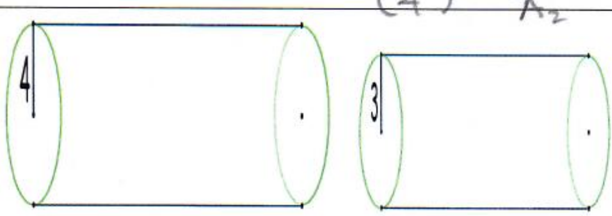
$\left(\frac{5}{4}\right)^2 = \frac{A_1}{A_2}$

Similarity ratio $\frac{5}{4}$ or 5:4

Ratio of surfacace area $\frac{25}{16}$ or 25:16

Ratio of volumes $\frac{125}{64}$ or 125:64

$r^3 = \frac{V_1}{V_2} \quad \left(\frac{5}{4}\right)^3 = \frac{V_1}{V_2}$



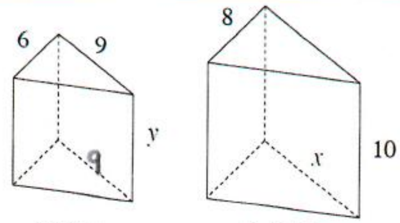
Similarity ratio $\frac{4}{3}$

Ratio of lateral area $\frac{16}{9}$ ← r^2 area

Ratio of volumes $\frac{64}{27}$ ← r^3

Ratio of circumferences $\frac{4}{3}$

9. The following pairs of figures are similar.



Solid A   Solid B

Determine x and y.

sim. ratio $= \frac{6}{8} = \frac{3}{4}$

$\frac{3}{4} = \frac{9}{x}$ $x = \frac{{}^{3}\cancel{9} \cdot 4}{\cancel{3}}$ $\boxed{x = 12}$

$\frac{3}{4} = \frac{y}{10}$ $y = \frac{{}^{5}\cancel{10} \cdot 3}{\cancel{4}_2} = \frac{15}{2}$

If the base area of figure A is 27 in² find the base area of B. (Hint: use the similarity ratio to determine. Do not calculate the area of the base using the dimensions.)

$r^2 = \frac{A_1}{A_2}$

$\left(\frac{3}{4}\right)^2 = \frac{27}{A_B} \Rightarrow \frac{9}{16} = \frac{27}{A_B}$ $A_B = \frac{\overset{3}{\cancel{27}} \cdot 16}{\cancel{9}}$

$A_B = 48 \text{ in}^2$

Find the volume of figure 1

Figure 1
Area = 18 in²

Volume = ?

Figure 2
Area = 98 in²

Volume = 1715 in ³

$r^2 = \frac{A_1}{A_2} = \frac{18}{98} = \frac{9}{49}$

$r = \frac{3}{7}$

$r^3 = \frac{V_1}{V_2}$

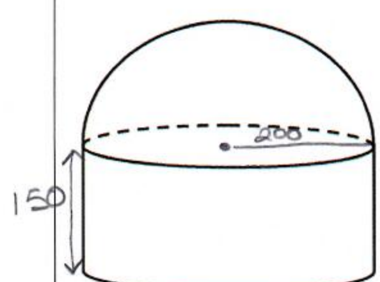
$\left(\frac{3}{7}\right)^3 = \frac{V_1}{1715}$

$\frac{27}{343} = \frac{V}{1715}$

$V_1 = \frac{27 \cdot 1715}{343}$

$V_1 = 135 \text{ in}^3$

Composite Figures



10. When considering new plans for a covered baseball stadium, Smallville looked into a design that used a cylinder with a dome in the shape of a hemisphere. The radius of the proposed cylinder is 200 feet and the height is 150 feet.

a. One of the concerns for the citizens of Smallville is the cost of heating the space inside the stadium for the fans. What is the volume of this stadium?

$V = Bh + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$= \pi r^2 h + \frac{2}{3}\pi r^3$

$= \pi (200)^2 150 + \frac{2}{3}\pi(200)^3$

$= 6000000\pi + 5333333.33\pi$

$\approx 35{,}604{,}716.74 \text{ ft}^3$

b. The citizens of Smallville are also interested in having the outside of the new stadium painted in green. Determine the surface area of the stadium that needs to be painted?

$SA = \frac{1}{2} \cdot 4\pi r^2 + 2\pi r h$

dome   ← cylinder without bases

$= 2\pi(200)^2 + 2\pi(200)150$

$= 80000\pi + 60000\pi$

$= 140000\pi$

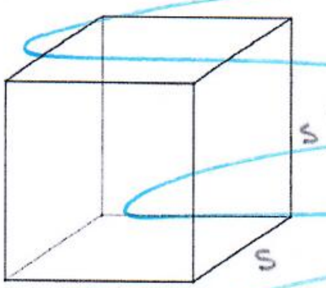
$\approx 439822.97 \text{ ft}^2$

More Solid Practice Problems

11

4.

The total area of the cube is 150 un^2.



$V = s^3$

$SA = s^2 \cdot 6$

$150 = 6s^2$

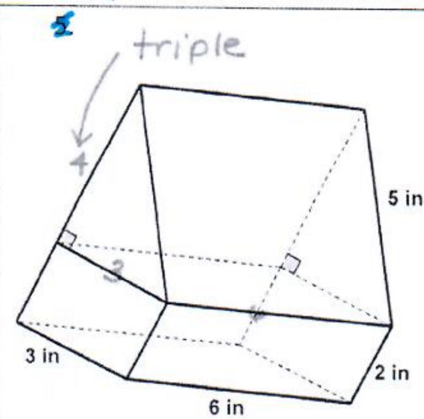
$25 = s^2$

$5 = s$

$V = s^3 = 5^3 = 125$

Find the volume $125\ u^3$

5



Surface Area $120\ in^2$

$SA = (3\cdot 2)2 + (6\cdot 2)2 + (3\cdot 6) +$

$2(\frac{1}{2}\cdot 3\cdot 4) + (6\cdot 5) + (6\cdot 4)$

$= 12 + 24 + 18 + 12 + 30 + 24$

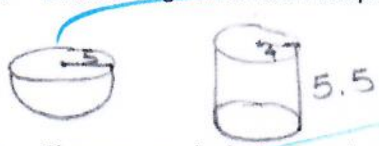
Volume $72\ in^3$

$V_{prism} = 3\cdot 2\cdot 6 = 36$

$V_{\Delta prism} = \frac{1}{2}\cdot 3\cdot 4\cdot 6 = 36$

6. Would a bowl (shaped as a hemisphere) with a diameter of 10" hold more or less water than a cylinder that had a diameter of 8" and a height of 5.5"?

a. Draw a diagram of each shape.



b. Show your calculations and state which solid would hold more water.

hemi

$V = \frac{1}{2}\cdot\frac{4}{3}\pi(5)^3$

$= \frac{2}{3}\pi(125)$

$= \frac{250}{3}\pi\ in^3$

$\approx 261.799\ in^3$

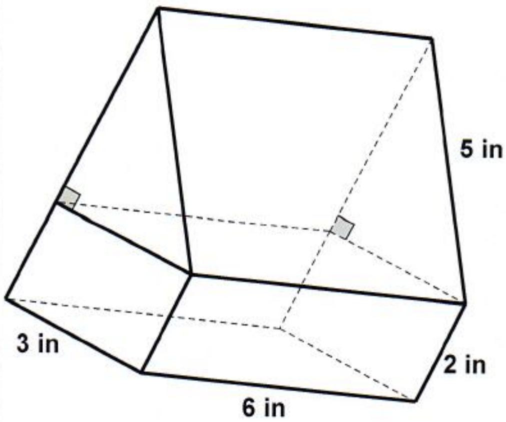
$V = \pi r^2 h$

$= \pi(4)^2 5.5$

$= 88\pi\ in^3$

$\approx 276.460\ in^3$

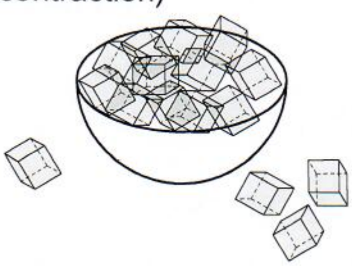
more water fits into the cylinder.



11. Surface area ______________________

Volume ______________________

12. There are 35 ice cubes in a bowl. The surface area of each ice cube is 54 cm^2. The bowl is the shape of a hemisphere with radius $7\frac{1}{2}$ cm^2. If all the ice cubes in the bowl melt, is the bowl big enough to hold all the water? (Perfect world: You do not need to consider expansion and contraction)



$54 = 6 \cdot s^2$

$9 = s^2$

$3 = s$

$V = 35 \cdot s^3$

$= 35 \cdot 27$

$= 945 \text{ cm}^3$

$V_{hemi} = \frac{1}{2} \cdot \frac{4}{3} \pi r^3$

$= \frac{2\pi}{3} (7.5)^3$

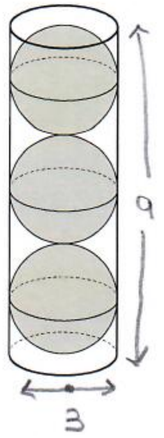
$= 883.13 \text{ cm}^3$

Volume of the water: 945 cm^3

Volume of the bowl: 883.13 cm^3

Does the water fit into the bowl? no

13. The figure shows a can of three tennis balls. The can is just large enough so that the tennis balls will fit inside with the lid on. The **diameter** of each tennis ball is 3 in. Round answers to the hundredth.



Volume of the can when empty:

$V = Bh$

$V = \pi(1.5)^2 9 \approx 63.62 \text{ in}^3$

Volume of space between the balls and the can:

V of cylinder − V of 3 balls

$V_B = 3 \cdot \frac{4}{3}\pi r^3 = 4\pi(1.5)^3 = 42.41$

$V_{\text{of space}} = 63.62 - 42.41 \approx 21.21 \text{ in}^3$

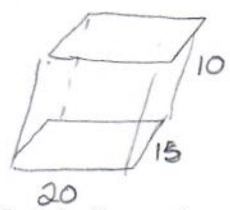
Surface area of the tennis balls:

$SA = 3 \cdot 4\pi r^2$

$= 12\pi(1.5)^2$

$\approx 84.82 \text{ in}^2$

14. A butterfly house at a local zoo is a rectangular prism with dimensions 20' × 15' × 10' and contains 625 butterflies. Sketch the prism on your paper.



a. What is the volume of the butterfly house?

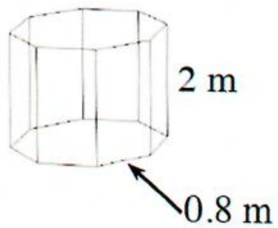
$V = 20 \cdot 15 \cdot 10 = 3000 \text{ ft}^3$

b. How many cubic feet of air is there for each butterfly? 3000 ft^3

c. **Density** is the quantity of something per unit measure, especially length, area, or volume. For example you might talk about the density of birds on a power wire (maybe a flock lands with 7 birds/meter), population density (the density of Singapore is 7301 people per square kilometer), or the mass density of an element (iron has density of 7.874 $\frac{g}{cm^3}$).

Assuming the butterflies are equally distributed inside the butterfly house, what is the density of butterflies? Explain.

$\frac{625}{3000} \approx 0.208 \frac{\text{butterflies}}{\text{ft}^3}$



15. A restaurant has a giant fish tank, shown below, in the shape of an octagonal prism.

a. Find the volume and surface area of the fish tank if the base is a regular octagon with side length 0.8 m and the height of the prism is 2 m.

b. What is the density of fish if there are 208 fish in the tank?

16. After a cylindrical birthday cake was sliced, a party guest received the slice at right. If her birthday cake originally had a diameter of 14 inches and a height of 6 inches, find the volume of her slice of cake.

[redacted]

[redacted]problems

~~112~~ ~~$V = 12\ u^3$~~

$SA = 42\ u^2$

a $\left(\frac{\text{scale}}{\text{factor}}\right)^3 = \frac{V}{V}$

$\left(\frac{4}{1}\right)^3 = \frac{V}{12}$ $\qquad \left(\frac{4}{1}\right)^2 = \frac{S}{42}$

$64 \cdot 12 = V$ $\qquad 42 \cdot 16 = SA$

$768\ u^3 = V$ $\qquad 672\ u^2 = SA$

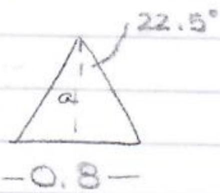
b $(r)^3 = \frac{1500}{12}$

$r^3 = 125$

~~$r = 5$~~

15

$V = Bh$



$\frac{360}{8} = 45$ $\qquad \tan 22.5 = a/0.4$

$45/2 = 22.5°$ $\qquad a = \frac{0.4}{\tan 22.5°}$

~ 0.9657

$B = \frac{1}{2}(0.8)(0.9657)\ 8$ ← # Δ's

$= 3.09$

$V = (3.09)(2)$

$\boxed{V = 6.18\ m^3}$

$SA = 2B + Ph$

$= 2(3.09) + (8 \cdot 0.8)\, 2$

$\boxed{SA = 18.98\ m^2}$

Density $= \#\text{fish}/V$

$= \frac{208 \text{ fish}}{6.18\ m^3}$

$\boxed{\text{Density} = 33.66\ \frac{\text{fish}}{m^3}}$

16

[redacted] $d = 14 \qquad r = 7 \qquad h = 6$

$$\underset{\text{slice}}{V \text{ of}} = \begin{pmatrix}\text{fraction} \\ \text{of whole} \\ \text{cake}\end{pmatrix} \overbrace{\begin{pmatrix}V \text{ of} \\ \text{whole} \\ \text{cake}\end{pmatrix}}^{\pi r^2 h}$$

$$= \frac{38}{360} \cdot \pi (7)^2\, 6$$

$$\approx 97.49 \text{ in}^3$$

OR same calculation

$$= \begin{pmatrix}\text{area of} \\ \text{sector}\end{pmatrix} \begin{pmatrix}h \text{ of} \\ \text{cake}\end{pmatrix}$$

$$= \left(\frac{38}{360} \cdot \pi 7^2\right) \cdot 6$$

$$\sim 97.49 \text{ in}^3$$