Unit 3 Study Guide

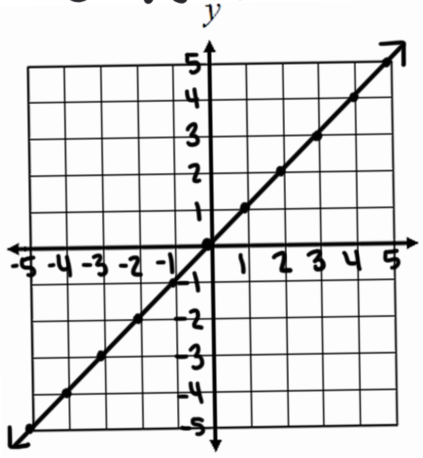
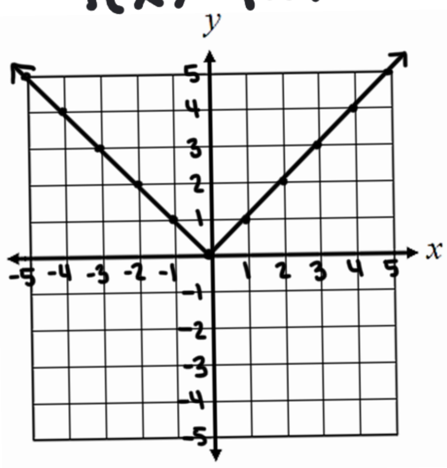
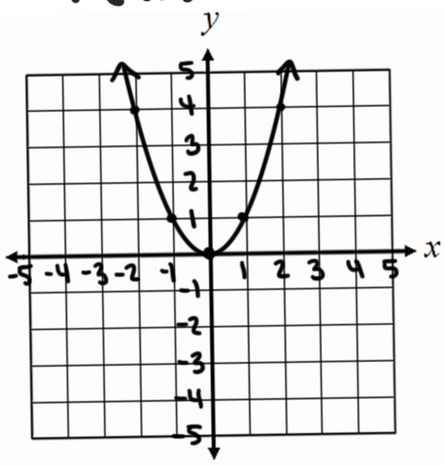
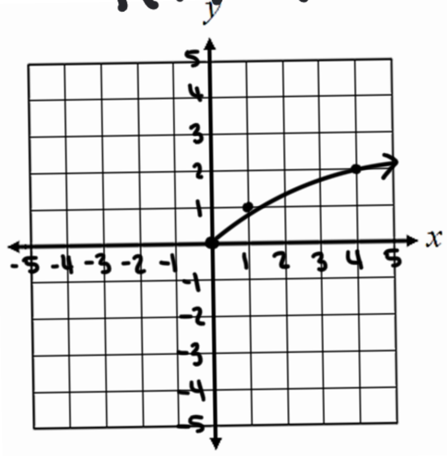
(Parent Functions & Transformations)

Name: ______________________

Date: ______________ **Block:** ________

Topic 1: Introduction to Parent Functions - Linear, Absolute Value, Quadratic, and Square Root

Identify each parent function, then graph:

Linear	Absolute Value	Quadratic	Square Root
$y = x$ or $f(x) = x$	$y = \|x\|$ $f(x) = \|x\|$	$y = x^2$ $f(x) = x^2$	$y = \sqrt{x}$ $f(x) = \sqrt{x}$
			

Topic 2: Transformation on Functions

Describe each transformation rule:

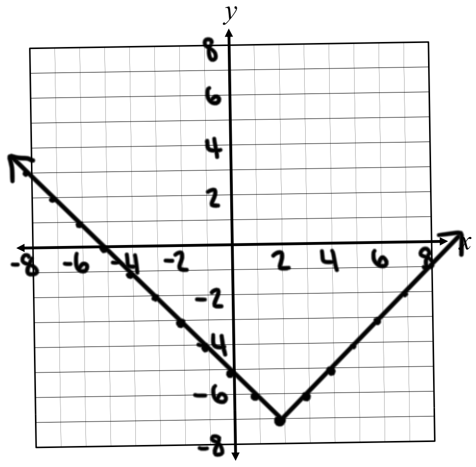
$f(x+h)$	left h units	$f(x-h)$	right h units
$f(x)+k$	up k units	$f(x)-k$	down k units
$a \cdot f(x)$ when $\|a\| > 1$	Vertically stretch by a factor of a.	$a \cdot f(x)$ when $\|a\| < 1$	vertically compressed by a factor of a
$-f(x)$	Reflect across x-axis.		

Topic 3: Absolute Value Functions

Vertex Form of an Absolute Value Function:

Graph each function using transformations. State the key features.

1. $f(x) = |x-2| - 7$



D: $(-\infty, \infty)$ R: $[-7, \infty)$

Vertex: $(2, -7)$

End Behavior:

As $x \to \infty$, $f(x) \to \infty$

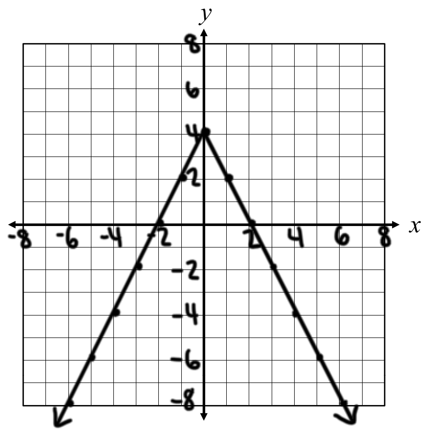
As $x \to -\infty$, $f(x) \to \infty$

Increasing Interval(s): $(2, \infty)$

Decreasing Interval(s): $(-\infty, 2)$

Remember: The increasing and decreasing intervals are the x values that cause the y values to increase or decrease.

2. $f(x) = -2|x| + 4$



D: $(-\infty, \infty)$ **R:** $(-\infty, 4]$

Vertex: $(0, 4)$

End Behavior:

As $x \to \infty$, $f(x) \to -\infty$

As $x \to -\infty$, $f(x) \to -\infty$

Increasing Interval(s): $(-\infty, 0)$

Decreasing Interval(s): $(0, \infty)$

The parent function of an absolute value equation is transfomed as described. Write the new equation in vertex form and identify the vertex.

3. Reflected about the x-axis, then shifted five units left.

$y = -|x+5|$

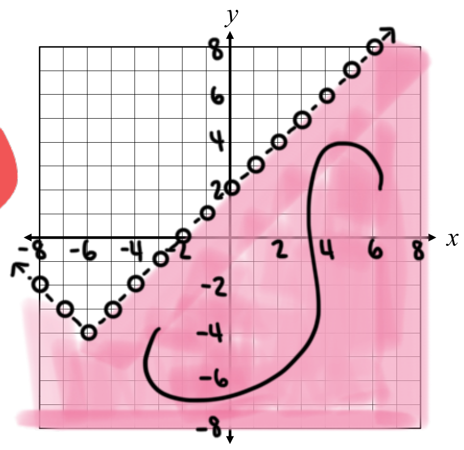
4. Vertically compressed by a factor of one third, then shifted one unit up and two units right.

$y = \frac{1}{3}|x-2| + 1$

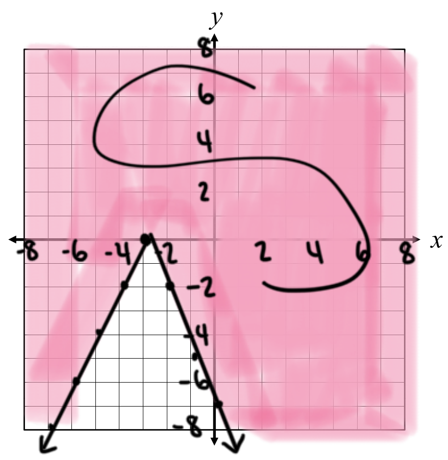
Graph each inequality.

5. $y < |x+6| - 4$

Dashed boundary



6. $y \geq -2|x+3|$



Topic 4: Quadratic Functions

Standard Form of a Quadratic Function:

$y = ax^2 + bx + c$

Vertex Form of a Quadratic Function:

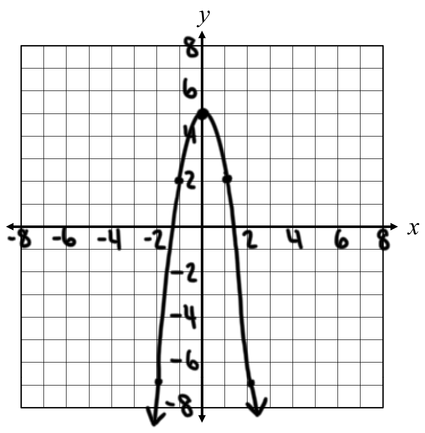
$y = a(x-h)^2 + k$

Graph the function. State the key features.

7. $f(x) = -3x^2 + 5$

$f(x) = -3(x-0)^2 + 5$

(0,5) vertex



D: $(-\infty, \infty)$ **R:** $(-\infty, 5]$

Vertex: $(0, 5)$

End Behavior:

As $x \to \infty$, $f(x) \to -\infty$

As $x \to -\infty$, $f(x) \to -\infty$

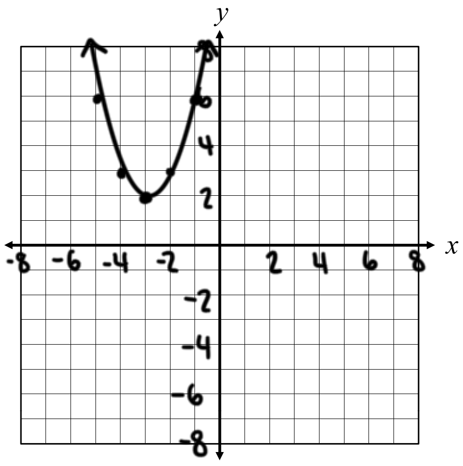
Increasing Interval(s): $(-\infty, 0)$

Decreasing Interval(s): $(0, \infty)$

two roots

8. $f(x)=(x+3)^2+2$

no real roots/zeros



D: $(-\infty,\infty)$ **R:** $[2,\infty)$

Vertex: $(-3,2)$

End Behavior:

As $x\to \infty$, $f(x)\to \infty$

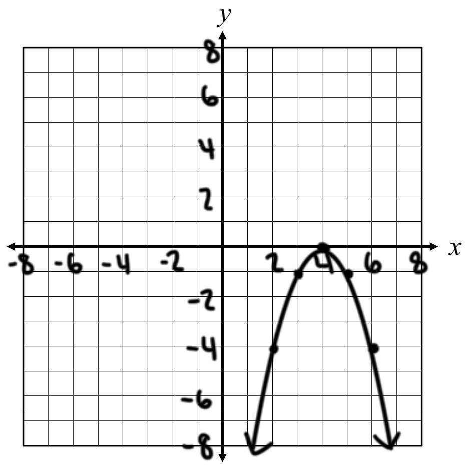
As $x\to -\infty$, $f(x)\to \infty$

Increasing Interval(s): $(-3,\infty)$

Decreasing Interval(s): $(-\infty,-3)$

9. $f(x)=-(x-4)^2$

1 root



D: $(-\infty,\infty)$ **R:** $(-\infty,0]$

Vertex: $(4,0)$

End Behavior:

As $x\to \infty$, $f(x)\to -\infty$

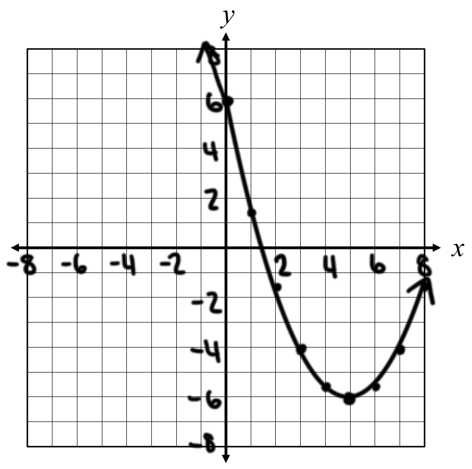
As $x\to -\infty$, $f(x)\to -\infty$

Increasing Interval(s): $(-\infty,4)$

Decreasing Interval(s): $(4,\infty)$

10. $f(x)=\frac{1}{2}(x-5)^2-6$

2 roots



D: $(-\infty,\infty)$ **R:** $[-6,\infty)$

Vertex: $(5,-6)$

End Behavior:

As $x\to \infty$, $f(x)\to \infty$

As $x\to -\infty$, $f(x)\to \infty$

Increasing Interval(s): $(5,\infty)$

Decreasing Interval(s): $(-\infty,5)$

The parent function of a quadratic equation is transfomed as described. Write the new equation in vertex form and identify the vertex.

11. Vertically stretched by a factor of four and shifted seven units right.

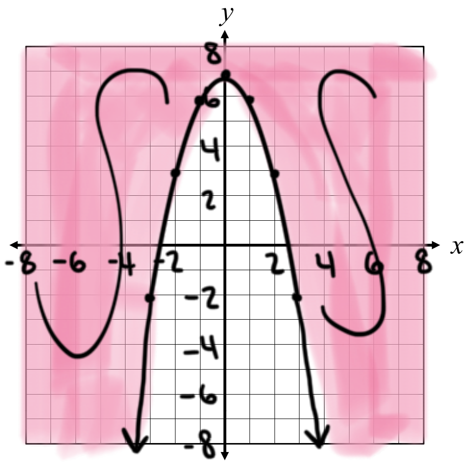
$y=4(x-7)^2$

12. Reflected across the x-axis, vertically compressed by a factor of one-half, then shifted four units left and six units down.

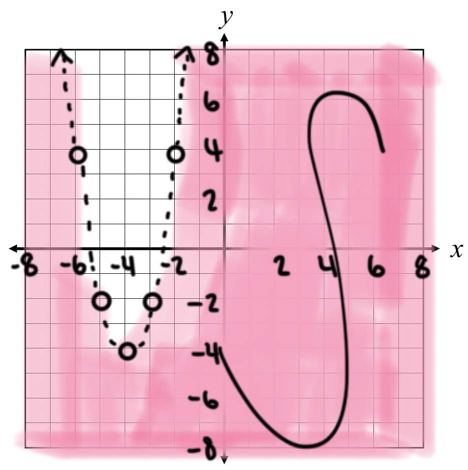
$y=-\frac{1}{2}(x+4)^2-6$

Graph each inequality.

13. $y\geq -x^2+7$



14. $y<2(x+4)^2-4$



Convert each function to vertex form. Then state the vertex.

15. $f(x) = x^2 - 16x + 64$

$f(x) = (x^2 - 16x + \underline{64}) + 64 - \underline{64}$

$f(x) = (x-8)^2$

vertex: $(8, 0)$

$\left(\frac{-16}{2}\right)^2$

$(-8)^2$

16. $f(x) = x^2 + 2x - 8$

$f(x) = (x^2 + 2x + \underline{1}) - 8 - \underline{1}$

$f(x) = (x+1)^2 - 9$

vertex: $(-1, -9)$

$\left(\frac{2}{2}\right)^2 = (1)^2$

17. $f(x) = -x^2 + 10x - 23$

$f(x) = -(x^2 - 10x + \underline{25}) - 23 - (\underline{25})(-1)$

$f(x) = -(x-5)^2 + 2$

vertex: $(5, 2)$

$\left(\frac{-10}{2}\right)^2$

$(-5)^2$

18. $f(x) = -2x^2 + 12x - 19$

$f(x) = -2(x^2 - 6x + \underline{9}) - 19 - (9)(-2)$

$f(x) = -2(x-3)^2 - 1$

vertex: $(3, -1)$

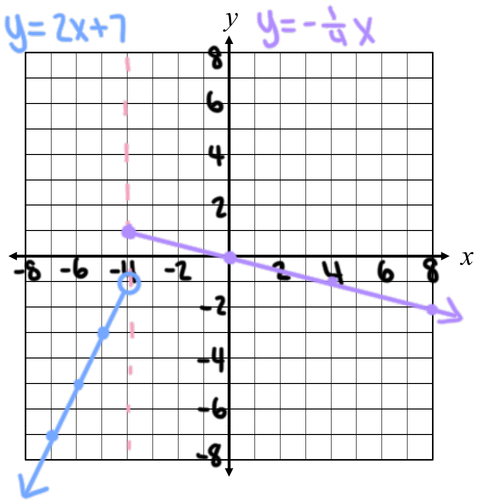
$\left(\frac{-6}{2}\right)^2 =$

$(-3)^2$

Topic 5: Piecewise Functions

Graph each function. State the domain and range.

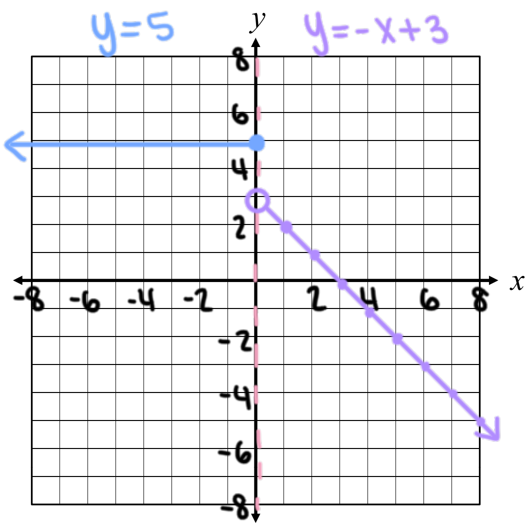
19. $f(x) = \begin{cases} \boxed{2x+7} & \text{if } x < -4 \\ \boxed{-\frac{1}{4}x} & \text{if } x \geq -4 \end{cases}$



Domain: $(-\infty, \infty)$

Range: $(-\infty, 1]$

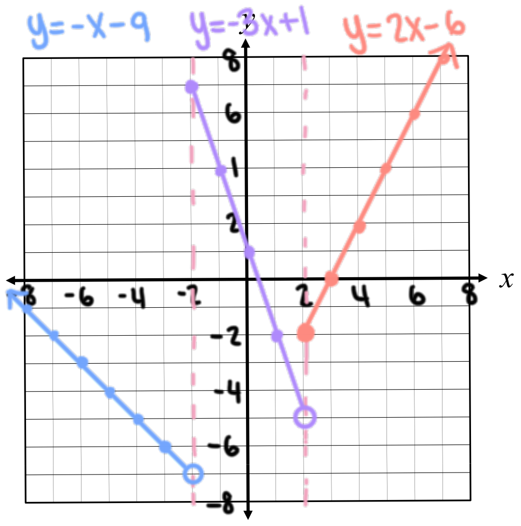
20. $p(x) = \begin{cases} \boxed{5} & \text{if } x \leq 0 \\ \boxed{-x + 3} & \text{if } x > 0 \end{cases}$



Domain: $(-\infty, \infty)$

Range: $(-\infty, 3) \cup \{5\}$

21. $h(x)=\begin{cases} \boxed{-x-9} & \text{if } x<-2 \\ \boxed{-3x+1} & \text{if } -2\le x<2 \\ \boxed{2x-6} & \text{if } x\ge 2 \end{cases}$



Domain: $(-\infty,\infty)$

Range: $(-7,\infty)$

Topic 6: Square Root Functions

Describe the transformations on each function compared to its parent function.

22. $f(x)=\frac{3}{4}\sqrt{x}+1$

vertically compressed by a factor of 3/4 and shifted up 1 unit.

23. $f(x)=2\sqrt{x+5}-8$

vertically stretched by a factor of 2, left 5 units and down 8 units.

24. The square root parent function is reflected about the *x*-axis, then shifted so that its endpoint is located at (-6, -2). Write an equation that represents this new function.

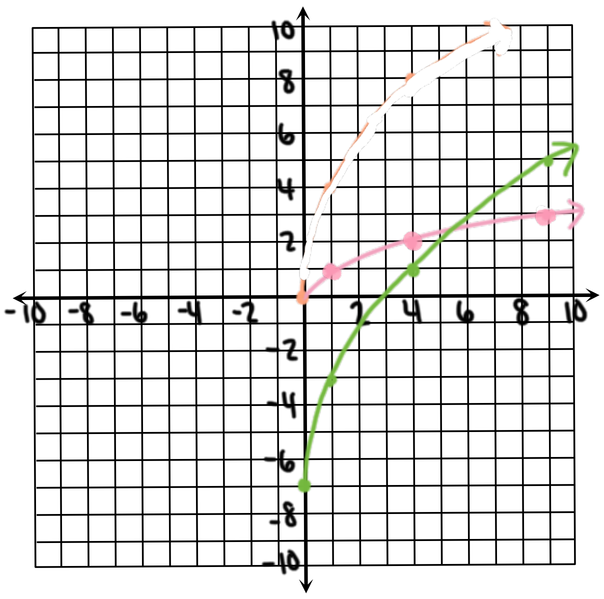
$y=-\sqrt{x+6}-2$

Graph each function and identify its key characteristics.

25. $\boxed{f(x)=4\sqrt{x}-7}$

$*y=\sqrt{x}$ parent

1 root | zero



D: $[0,\infty)$ R: $[-7,\infty)$

Endpoint: $(0,-7)$

End Behavior:

As $x\to \infty$, $f(x)\to \infty$

As $x\to 0^+$, $f(x)\to -7$

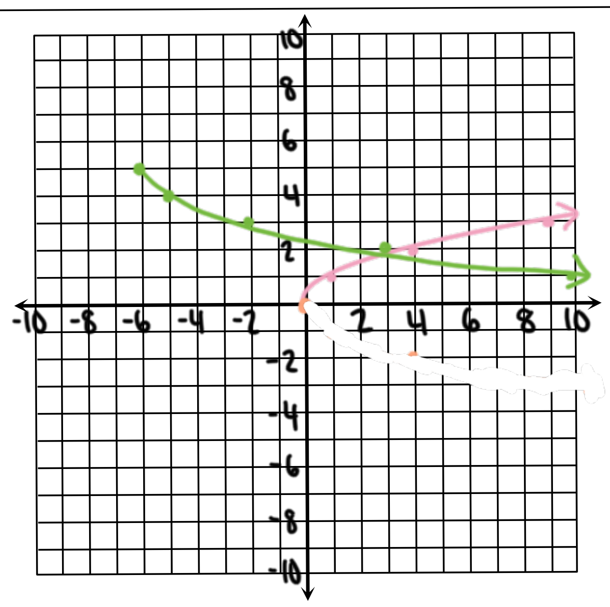
Increasing Interval(s): $[0,\infty)$

Decreasing Interval(s): N/A

26. $\boxed{f(x)=-\sqrt{x+6}+5}$

$*y=\sqrt{x}$ parent

1 root | zero



D: $[-6,\infty)$ R: $(-\infty,5]$

Endpoint: $(-6,5)$

End Behavior:

As $x\to \infty$, $f(x)\to -\infty$

As $x\to -6^+$, $f(x)\to 5$

Increasing Interval(s): N/A

Decreasing Interval(s): $[-6,\infty)$