

Name:

Date:

Period:

6.8 Solving Polynomials with Real

Complex Notes

Review Factor practice h.o.
 6.4 factor by grouping
 6.5 solving by factoring
 6.7 solving with div.

Irrational
Zeros

1. $f(x) = 4x^4 - 33x^2 + 8$

factors

#9 6.4
Factoring
Prac.

$$\begin{array}{r} 32x^4 \\ -32x^2 \quad -1x^2 \\ -33x^2 \end{array}$$

$$\begin{aligned} 4x^4 - 32x^2 - 1x^2 + 8 &= f(x) \\ 4x^2(x^2 - 8) - 1(x^2 - 8) &= \\ (4x^2 - 1)(x^2 - 8) &= \\ (2x - 1)(2x + 1)(x^2 - 8) &= \\ 2x - 1 = 0 & \quad 2x + 1 = 0 & \quad x^2 - 8 = 0 \quad \text{solve for zeros} \\ 2x = 1 & \quad 2x = -1 & \quad x^2 = 8 \\ x = \frac{1}{2} & \quad x = -\frac{1}{2} & \quad x = \pm\sqrt{8} = \pm 2\sqrt{2} \end{aligned}$$

Zeros: $x = \pm \frac{1}{2}, \pm 2\sqrt{2}$

2. $f(x) = (x^3 - 2x^2) - (18x + 36)$ factors

$$\begin{aligned} &= x^2(x - 2) - 18(x - 2) \\ &= (x^2 - 18)(x - 2) \\ &= 2 \text{ does not factor} \end{aligned}$$

$$\begin{aligned} x^2 - 18 = 0 & \quad x - 2 = 0 \\ x^2 = 18 & \quad x = 2 \\ x = \pm\sqrt{18} = \pm 3\sqrt{2} \end{aligned}$$

Zeros: $x = \{\pm 3\sqrt{2}, 2\}$

3. $f(x) = 2x^4 - 5x^3 - 4x^2 + 15x - 6; x = 2$

Does not factor

$$\begin{array}{r} 2 \overline{) 2 \quad -5 \quad -4 \quad 15 \quad -6} \\ \underline{4 \quad -2 \quad -12 \quad 6} \\ 2 \quad -1 \quad -6 \quad 3 \quad 0 \end{array}$$

6.7 notes
find zeros by
div.

$$f(x) = (2x^3 - x^2 - 6x + 3)(x - 2)$$

factors: grouping

$$(2x^3 - x^2) - (6x + 3)$$

$$x^2(2x - 1) - 3(2x - 1)$$

$$(x^2 - 3)(2x - 1)$$

$$f(x) = (x^2 - 3)(2x - 1)(x - 2)$$

$$\begin{aligned} x^2 - 3 = 0 & \quad 2x - 1 = 0 & \quad x - 2 = 0 \\ x^2 = 3 & \quad 2x = 1 & \\ x = \pm\sqrt{3} & \quad x = \frac{1}{2} & \quad x = 2 \end{aligned}$$

Zeros

$$x = \{\pm\sqrt{3}, \frac{1}{2}, 2\}$$

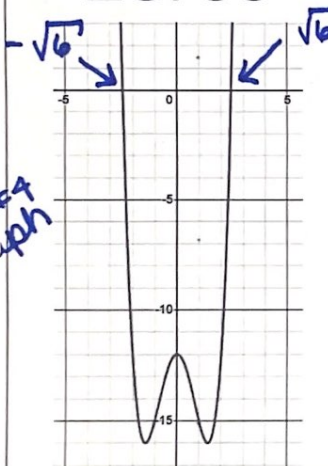
The Fundamental Theorem of Algebra

A polynomial function of degree n has at least

one zero in the complex system.

* Remember: Any real number is also a complex number

Complex Zeros



Two complex (do not cross x axis)

6.2 shapes and key features

4. $f(x) = x^4 - 4x^2 - 12$

$(x^2)^2$
 $f(x) = (x^2 - 6)(x^2 + 2)$
 cannot factor

$x^2 - 6 = 0$ $x^2 + 2 = 0$
 $x^2 = 6$ $x^2 = -2$
 $x = \pm\sqrt{6}$ $x = \pm i\sqrt{2}$

Two real (irrational) and two complex

5. $f(x) = 2x^5 - 5x^4 - 2x + 5$

$= x^4(2x - 5) - 1(2x - 5)$

$= (x^4 - 1)(2x - 5)$

$= (x^2 - 1)(x^2 + 1)(2x - 5)$

$= (x - 1)(x + 1)(x^2 + 1)(2x - 5)$ factored completely

or
 $x^2 - 1 = 0$ $x^2 + 1 = 0$ $2x - 5 = 0$
 $x^2 = 1$ $x^2 = -1$ $2x = 5$
 $x = \pm 1$ $x = \pm i$ $x = \frac{5}{2}$

Zeros: $x = \pm 1, \pm i, \frac{5}{2}$

6. $f(x) = x^5 - 3x^4 - x^3 + 3x^2 - 2x + 6; x = 3$

$$\begin{array}{r|rrrrrr} 3 & 1 & -3 & -1 & 3 & -2 & 6 \\ & & 3 & 0 & -3 & 0 & -6 \\ \hline & 1 & 0 & -1 & 0 & -2 & 0 \end{array}$$

$f(x) = (x - 3)(x^4 - x^2 - 2)$

$f(x) = (x - 3)(x^2 - 2)(x^2 + 1)$

$x = 3$ $x^2 = 2$ $x^2 + 1 = 0$
 $x = \pm\sqrt{2}$ $x^2 = -1$
 $x = \pm i$

Zeros $x = \{\pm\sqrt{2}, 3, \pm i\}$