| Name: | Date: | Period: |
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# 6.7 Solving Polynomials with Real Zeros Notes

## Warm up

Is 9 a factor of 1278?

Yes or No yes

Explain how you determined. 9 divides into 1278 with no remainder

## Remainder Theorem

- If a polynomial function $f(x)$ is divided by $(x-c)$, then its remainder is $f(c)$.

## Examples

Use the Remainder Theorem to evaluate $f(x)$ at $c$.

1. $f(x) = x^4 + 8x^3 + 12x^2 - 7x - 14; c = -3$

$f(-3) = (-3)^4 + 8(-3)^3 + 12(-3)^2 - 7(-3) - 14$

$= 81 + 8(-27) + 12(9) + 21 - 14$

$= 81 - 216 + 108 + 7$

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$f(-3) = -20$

2. $f(x) = -2x^5 + 7x^4 + 2x^3 - x + 8; c = 4$

$f(4) = -2(4)^5 + 7(4)^4 + 2(4)^3 - 4 + 8$

$= -2(1024) + 7(256) + 2(64) + 4$

$= -2048 + 1792 + 128 + 4$

$f(4) = -124$

## Factor Theorem

Given a polynomial function $f(x)$, $(x-c)$ is a factor of $f(x)$ if and only if

$f(c) = 0$

## Factoring & Finding Zeros

Directions: Use the factor theorem to verify (x-5) is a factor. Then find all the zeros.

3. $f(x) = x^3 - 2x^2 - 19x + 20;\ (x-5)$

$c = 5$

$f(5) = (5)^3 - 2(5)^2 - 19(5) + 20$

$= 125 - 50 - 95 + 20$

$f(5) = 0$ ✓

$$\begin{array}{r|rrrr} 5 & 1 & -2 & -19 & 20 \\ & & 5 & 15 & -20 \\ \hline & 1 & 3 & -4 & 0 \end{array}$$

$f(x) = (x-5)(x^2+3x-4)$

$= (x-5)(x+4)(x-1)$

$x-5=0 \quad x+4=0 \quad x-1=0$

$x = \{5, -4, 1\}$

zeros

| | |
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| $f(-4) = 2(-4)^3 - 7(-4)^2 - 53(-4) + 28$<br>$= 2(-64) - 7(16) + 212 + 28$<br>$= -128 - 112 + 212 + 28$<br>$f(-4) = 0$ | 4. $f(x) = 2x^3 - 7x^2 - 53x + 28;\ (x+4)$ $c = -4$<br>$-4 \,\rfloor\ 2 \quad -7 \quad -53 \quad +28$<br>$\quad\quad -8 \quad 60 \quad -28$<br>$\quad 2 \quad -15 \quad 7 \quad 0$<br>$f(x) = (x+4)(2x^2 - 15x + 7)$<br>$f(x) = (x+4)(2x-1)(x-7)$<br>$0 = (x+4)$ $0 = (2x-1)$ $0 = x-7$<br>$x = \{-4, \frac{1}{2}, 7\}$ |
| Rational Zero Theorem | The **Rational Zero Theorem** can be used to determine all possible *rational* zeros of a polynomial function.<br>If $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has integer coefficients, then every rational zero of the function has the following form:<br>$\frac{p}{q} = \frac{\text{factors of the constant } (a_0)}{\text{factors of the leading coefficient } (a_n)}$ |
| Finding Rational Zeros | 1. List all possible rational zeros using the **Rational Zero Theorem.**<br>2. Test the possible zeros using the **Remainder Theorem.**<br>3. When you find a zero that works, use synthetic division to divide it out. Then find the remaining zeros. |
| We will check 1, -1 only for the notes.<br>$f(-1) = (-1)^2 - 21(-1) + 20$<br>$= -1 + 21 + 20$<br>$= 40$ not a zero<br>$f(1) = (1)^3 - 2(1) + 20$<br>$= 1 - 21 + 20$<br>$= -21 + 20$<br>$= 0$ ✓<br>root | 5. $f(x) = x^3 - 21x + 20$<br>All possible zeros<br>$\frac{20}{1}$: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$<br>$1 \,\rfloor\ 1 \quad 0 \quad -21 \quad 20$<br>$\quad\quad 1 \quad 1 \quad -20$<br>$\quad 1 \quad 1 \quad -20 \quad 0$<br>Linear factor, quad. factor<br>$f(x) = (x-1)(x^2 + x - 20)$ factored form<br>$= (x-1)(x+5)(x-4)$<br>$0 = x-1$ $0 = x+5$ $0 = x-4$<br>$x = \{-5, 1, 4\}$ |