

5.7 Exponential base e and Natural Logs Notes

<p>Warm up</p> <p>$\log_{10} x$ $\log x$ common Log</p>	<p>Solve $4^{x+2} = 45$</p> <p>$\log 4^{x+2} = \log 45$ $(x+2)\log 4 = \log 45$ $x+2 = \frac{\log 45}{\log 4}$ $x = \frac{\log 45}{\log 4} - 2 \approx 0.746$</p> <p>Rewrite the Log equation into an exponential equation.</p> <p>$\log x = 7$ $10^7 = x$</p>
<p>Irrational number</p> <p>e</p> <p><u>2.7118281...</u></p>	<p>The number e is used in many real-world applications, such as <u>economics</u>, <u>statistics</u>, and <u>science</u> that involve exponential growth and decay.</p>
<p>Example of exponential model</p>	<p>Newton's Law of Cooling</p> $T(t) = T_s + (T_0 - T_s)e^{-kt}$ <p>t = time taken for the cooling $T(t)$ = the temperature of the given object at time t T_s = the surrounding temperature T_0 = the initial temperature of the object k = decay constant</p>
<p>Log of base e</p> <p>Natural Log</p>	<p>$e^p = n$</p> <p>Rewritten into log form</p> <p>$\log_e n = p$</p> <p><u>$\log_e n = p$</u> is written as <u>$\ln(n) = p$</u></p>

Directions. Rewrite each exponential into log form and each log into exponential form.

1. $e^x = 24$ $\log_e 24 = x$ $\rightarrow \ln 24 = x$	2. $e^{x+5} = 72$ $\ln 72 = x+5$	3. $\ln x = 58$ $e^{58} = x$	4. $\ln(x-9) = 32$ $e^{32} = x-9$
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Directions: Solve the following equations. See notes 5.5 + 5.6

5. $2 \ln k = \ln(2k+15)$ $\ln k^2 = \ln(2k+15)$ $k^2 = 2k+15$ $k^2 - 2k - 15 = 0$ $(k-5)(k+3) = 0$ $k=5$ $k = 5, -3$ Check $k = -3$ extraneous	6. $\ln 72 - \ln 4 = \ln 6 + \ln(a-2)$ $\ln\left(\frac{72}{4}\right) = \ln[6(a-2)]$ $\ln(18) = \ln(6a-12)$ $18 = 6a-12$ $30 = 6a$ $a = 5$ Check \checkmark
7. $\ln x - \ln 9 = 7$ $\ln\left(\frac{x}{9}\right) = 7$ $e^7 = \frac{x}{9}$ $x = 9 \cdot e^7$ $x \approx 9869.698$	8. $\ln 8x = 2$ $e^2 = 8x$ $x = \frac{e^2}{8}$ $x \approx 0.924$
9. $2e^{c-9} + 3 = 87$ $2e^{c-9} = 84$ $e^{c-9} = 42$ Take \ln of both sides $\ln e^{c-9} = \ln 42$ $\log_e e^{c-9} =$ $c-9 = \ln 42$ $c = \ln 42 + 9$ $c \approx 12.738$	10. $e^{x+3} - 6 = 24$ $e^{x+3} = 30$ $\ln e^{x+3} = \ln 30$ $x+3 = \ln 30$ $x = \ln 30 - 3$ $x \approx 0.401$

$\log_3 3^2$ simplifies to 2