4-24. Solve each of the following equations using any method. Be sure to check your solutions.

$2|3x+4| = 22$

$3x+4 = 11$

a. $-3\sqrt{2x-5}+7=-8$

[$x=15$]

$-3\sqrt{2x-5} = -15$

$\sqrt{2x-5} = 5$

$2x-5 = 25$

$2x = 30$

b. $2|3x+4|-10=12$

[$x=\frac{7}{3}$ or $x=-5$]

$3x+4 = 11$

$3x = 7$

$x = 7/3$

$3x+4 = -11$

$3x = -15$

$x = -5$

4-25. Ted needs to find the point of intersection for the lines $y=18x-30$ and $y=-22x+50$. He takes out a piece of graph paper and then realizes that he can solve this problem without graphing. Explain how Ted is going to accomplish this, and then find the point of intersection. **[The lines intersect at the point (2, 6). Ted will solve the system algebraically by setting $18x-30=-22x+50$.]**

4-26. Consider the arithmetic sequence $2,\ a-b,\ a+b,\ 35,\ \ldots$ Find a and b. **[$a=18.5$, $b=5.5$]**

$t(n)$ 2, $\overset{2+11}{13}$, 24, 35

n 1, 2, 3, 4

$d = \frac{35-2}{4-1} = \frac{33}{3} = 11$

see below

4-27. Solve the following equations. Be sure to check your answers for any extraneous solutions.

a. $\sqrt{2x-1}-x=-8$ **[$x=13$, $x=5$ is extraneous]** below

b. $\sqrt{2x-1}-x=0$ **[$x=1$]**

$\sqrt{2x-1} = x$

$2x-1 = x^2$

$x^2-2x+1=0$

$(x-1)(x-1)=0$

$x=0$

$\sqrt{2(1)-1} - 1 \overset{?}{=} 0$

$1 - 1 = 0$

$0 = 0$ ✓

4-28. Find the value of x.

a. 15, 39, x

[$x=36$]

$15^2 + x^2 = 39^2$

$x = \sqrt{1296}$

$x = 36$

b. 10, 30, x

[$x=20\sqrt{2}$ or $x\approx 28.28$]

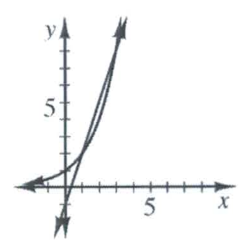
$10^2 + x^2 = 30^2$

$x^2 = \sqrt{800}$

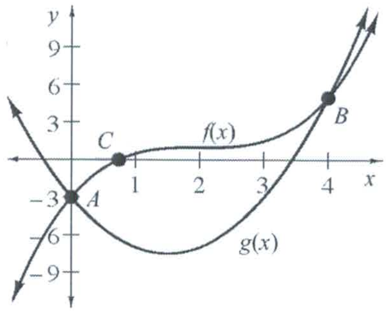
$x = \sqrt{400 \cdot 2}$

$x = 20\sqrt{2}$

4-29. Solve $3x-1=2^x$ graphically. Could you solve this equation algebraically? Explain. **[See graph at right. $x=1$ and $x=3$; No]**



4-30. Consider the graphs of $\boxed{f(x)=\frac{1}{2}(x-2)^3+1}$ and $\boxed{g(x)=2x^2-6x-3}$ at right.



a. Write an equation that you could solve using points A and B. What are the solutions to your equation? Substitute them into your equation to show that they work.
[$\frac{1}{2}(x-2)^3+1=2x^2-6x-3$, $x=0$ or $x=4$]

b. Are there any solutions to the equation in part (a) that do not appear on the graph? Explain. **[$x=6$ is also a solution.]**
look at the tables on your calculator

c. Write an equation that you could solve using point C. What does the solution to your equation appear to be? Again, substitute your solution into the equation. How close was your estimate? **[$\frac{1}{2}(x-2)^3+1=0$, $x\approx 0.74$]**

on x-axis
Pt C is only on f(x) ∴ $y=0$
$0=\frac{1}{2}(x-2)^3+1$
$-1=\frac{1}{2}(x-2)^3$
$-2=(x-2)^3$
$\sqrt[3]{-2}=x-2$
$-1.26=x-2$
$x\sim 0.74$

d. What are the domains and ranges of $f(x)$ and $g(x)$?
[Domain and range of $f(x)$: all real numbers, domain of $g(x)$: all real numbers, range of $g(x)$: $y\geq -7.5$]

4-31. Solve each of the following equations using any method.

a. $2(x+3)^2-5=-5$
[$x=-3$]

$2(x+3)^2=0$
$(x+3)^2=0$
$x+3=0$
$x=-3$

b. $3(x-2)^2+6=9$
[$x=1$ or $x=3$]

$3(x-2)^2=3$
$(x-2)^2=1$
$x-2=\pm 1$
$x=2+1 \quad x=2-1$
$x=3, 1$

c. $|2x-5|-6=15$
[$x=-8$ or $x=13$]

$|2x-5|=21$
$2x-5=21$ $2x-5=-21$
$2x=26$ $2x=-16$
$x=13$ $x=-8$

d. $3\sqrt{5x-2}+1=7$
[$x=1.2$]

$3\sqrt{}=6$
$\sqrt{5x-2}=2$
$5x-2=4$
$5x=6$
$x=6/5$

4-32. Solve each of the following equations for the indicated variable.

a. $5x-3y=12$ for y
[$y=\frac{5}{3}x-4$]

$-3y=-5x+12$
$y=\frac{5}{3}x-4$

b. $F=\frac{Gm_1m_2}{r^2}$ for m_2
[$m_2=\frac{Fr^2}{Gm_1}$]

$Fr^2=Gm_1m_2$
$\frac{Fr^2}{Gm_1}=m_2$

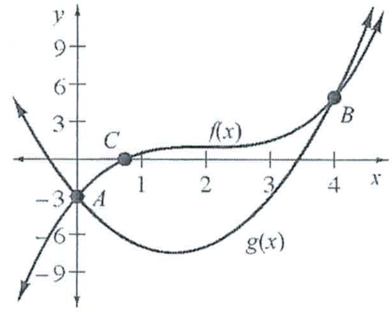
c. $E=\frac{1}{2}mv^2$ for m
[$m=\frac{2E}{v^2}$]

$E=\frac{1}{2}mv^2$
$2E=mv^2$
$\frac{2E}{v^2}=m$

d. $(x-4)^2+(y-1)^2=10$ for y
[$y=\pm\sqrt{10-(x-4)^2}+1$]

$(y-1)^2=10-(x-4)^2$
sq root
$y-1=\pm\sqrt{10-(x-4)^2}$
$y=\pm\sqrt{10-(x-4)^2}+1$

4-30. Consider the graphs of $\boxed{f(x)=\frac{1}{2}(x-2)^3+1}$ and $\boxed{g(x)=2x^2-6x-3}$ at right.



a. Write an equation that you could solve using points A and B. What are the solutions to your equation? Substitute them into your equation to show that they work.
[$\frac{1}{2}(x-2)^3+1=2x^2-6x-3$, $x=0$ or $x=4$]

b. Are there any solutions to the equation in part (a) that do not appear on the graph? Explain. **[$x=6$ is also a solution.]**

look at the tables on your calcula

c. Write an equation that you could solve using point C. What does the solution to your equation appear to be? Again, substitute your solution into the equation. How close was your estimate? **[$\frac{1}{2}(x-2)^3+1=0$, $x\approx 0.74$]**

on x-axis

Pt C is only on f(x) ∴
$y=0$
$0=\frac{1}{2}(x-2)^3+1$
$-1=\frac{1}{2}(x-2)^3$
$-2=(x-2)^3$
$\sqrt[3]{-2}=x-2$
$-1.26=x-2$
$x\sim 0.74$

d. What are the domains and ranges of $f(x)$ and $g(x)$?
[Domain and range of $f(x)$: all real numbers, domain of $g(x)$: all real numbers, range of $g(x)$: $y\geq -7.5$]

4-31. Solve each of the following equations using any method.

a. $2(x+3)^2-5=-5$
[$x=-3$]

$2(x+3)^2=0$
$(x+3)^2=0$
$x+3=0$
$x=-3$

b. $3(x-2)^2+6=9$
[$x=1$ or $x=3$]

$3(x-2)^2=3$
$(x-2)^2=1$
$x-2=\pm 1$
$x=2+1$ $x=2-1$
$x=3,1$

c. $|2x-5|-6=15$
[$x=-8$ or $x=13$]

$|2x-5|=21$
$2x-5=21$ $2x-5=-21$
$2x=26$ $2x=-16$
$x=13$ $x=-8$

d. $3\sqrt{5x-2}+1=7$
[$x=1.2$]

$3\sqrt{}=6$
$\sqrt{5x-2}=2$
$5x-2=4$
$5x=6$
$x=6/5$

4-32. Solve each of the following equations for the indicated variable.

a. $5x-3y=12$ for y
[$y=\frac{5}{3}x-4$]

$-3y=-5x+12$
$y=\frac{5}{3}x-4$

b. $F=\frac{Gm_1m_2}{r^2}$ for m_2
[$m_2=\frac{Fr^2}{Gm_1}$]

$Fr^2=Gm_1m_2$
$\frac{Fr^2}{Gm_1}=m_2$

c. $E=\frac{1}{2}mv^2$ for m
[$m=\frac{2E}{v^2}$]

$E=\frac{1}{2}mv^2$
$2E=mv^2$
$\frac{2E}{v^2}=m$

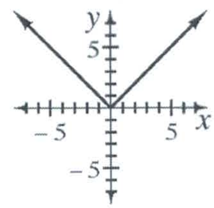
d. $(x-4)^2+(y-1)^2=10$ for y
[$y=\pm\sqrt{10-(x-4)^2}+1$]

$(y-1)^2=10-(x-4)^2$
sq root
$y-1=\pm\sqrt{10-(x-4)^2}$
$y=\pm\sqrt{10-(x-4)^2}+1$

4-33. Paul states that $(a+b)^2$ is equivalent to a^2+b^2. Joyce thinks that something is missing. Help Joyce show Paul that the two expressions are not equivalent. Explain using at least two different approaches: diagrams, algebra, numbers, or words.
[$(a+b)^2 = a^2+2ab+b^2$, substitute numbers, etc.]

$(a+b)(a+b) = a^2 + ab + ba + b^2$
$= a^2 + 2ab + b^2$

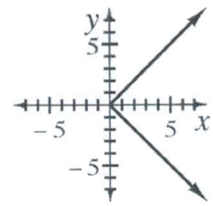
4-34. Graph each of the following equations. (Keep the graphs handy, because you will need them for your homework for Lesson 4.1.3.)
[a: See graph at top right. b: See graph at bottom right.]



a. $y=|x|$

b. $|y|=x$

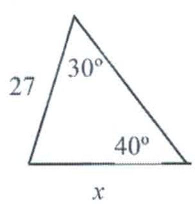
c. How are the two graphs similar? How are they different?
[Graph (b) is similar to graph (a), but is rotated 90° clockwise.]



d. What are the domain and range of each relation? **[(a) D: all real numbers, R: $y \geq 0$; (b) D: $x \geq 0$, R: all real numbers]**

4-35. Find the value of x.

a.



[21.00]

Law of Sines

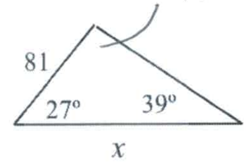
$\frac{\sin 40°}{27} = \frac{\sin 30°}{x}$

$x = \frac{27 \cdot \sin 30°}{\sin 40°}$

$x \sim 21.0$

b.

114° (Δ Sum)



[117.58]

$\frac{\sin 114}{x} = \frac{\sin 39}{81}$

$x = \frac{81 \cdot \sin 114°}{\sin 39°}$

$x \sim 117.6$

Lesson 4.1.3 How many solutions are there?

Finding Multiple Solutions to Systems of Equations

Lesson Objective: Students will solve systems of linear and non-linear equations using multiple strategies. Students will determine the number of solutions for systems and interpret solutions graphically.

Mathematical Practices: reason abstractly and quantitatively, attend to precision, look for and make use of structure

Length of Activity: One day (approximately 50 minutes)

Core Problems: Problems 4-36 and 4-37

Materials: None

Suggested Lesson Activity:

Introductory Discussion: Start this lesson by leading a short discussion about what students learned in Lessons 4.1.1 and 4.1.2. You could do this as a Dyad where students pair up, either with a teammate or by using proximity partners. One person talks about what they know about solving equations for 30 seconds. Then they switch roles. Have students recognize that they have developed several solving strategies and that they have focused both on one-variable and two-variable equations. In addition, students have learned how to check a solution, identify extraneous solutions, and find a solution using a graph. Explain that in this lesson, they will focus on the solutions for systems of equations in two variables, looking at both linear and non-linear relationships.

Solving Systems: Start teams on problems 4-36 and 4-37. Remind them that there are many different strategies that can be useful and to listen to the ideas of their teammates before embarking on a strategy for each system. The more solving strategies that emerge during this lesson, the more strategies students will be comfortable using later with systems of three equations and three unknowns in Section 6.1. You could consider using a Pairs Check on this problem. That would allow you the opportunity to do formative assessment as you circulate and listen to student explanations.

As you circulate, ask students to explain what the solution tells them about the graph of the system. For example, students should recognize that since the system in part (a) of problem 4-36 has no solution, the lines are parallel, while the lines in part (d) must coincide since the solution includes all points on the line $y = 2x - 5$. Once a team recognizes that the parabola and line in part (b) intersect only once, you can ask, "*What would that look like?*"

If teams get stuck on part (a) of problem 4-37, it might be helpful to have them think about which representation will help them answer the question or ask, "*Would visualizing the graphs make answering the question easier?*" Expect some teams to be challenged when trying to