

Name:

Date:

Period:

## 2.7 Systems with 3 Variables Notes

$$\begin{array}{l} \textcircled{1} \quad 2x + 3y + 3z = 6 \\ \textcircled{2} \quad 6x - 3y + 4z = 12 \\ \textcircled{3} \quad 2x - 3y + 2z = 6 \end{array}$$

A. Notice that the first two equations could be combined to form the new equation  $8x + 7z = 18$ .

How was this accomplished?

Adding  $\textcircled{1} + \textcircled{2}$

$$\begin{array}{r} \textcircled{1} \quad 2x + 3y + 3z = 6 \\ \textcircled{2} \quad 6x - 3y + 4z = 12 \\ \hline 8x + 7z = 18 \end{array} \quad \textcircled{A}$$

B. Now there is an equation with only  $x$  and  $z$ , find another equation with only  $x$  and  $z$  to be able to solve the system. Choose a different pair of equations to combine and find a way to eliminate  $y$  so that the new equation only has  $x$  and  $z$ .

$$\begin{array}{r} \textcircled{1} \quad 2x + 3y + 3z = 6 \\ \textcircled{3} \quad 2x - 3y + 2z = 6 \\ \hline \textcircled{B} \quad 4x + 5z = 12 \end{array}$$

C. Then solve the system of two equations to find  $x$  and  $z$ .

$$\begin{array}{l} \textcircled{A} \quad 8x + 7z = 18 \\ \textcircled{B} \quad 4x + 5z = 12 \\ \hline \textcircled{A} \quad 8x + 7z = 18 \\ \textcircled{B} \quad -8x - 10z = -24 \\ \hline -3z = -6 \end{array} \quad \begin{array}{l} \rightarrow z = 2 \\ \textcircled{B} \quad 4x + 5(2) = 12 \\ 4x + 10 = 12 \\ 4x = 2 \\ x = \frac{2}{4} = \frac{1}{2} \end{array}$$

D. Now for which variable do you still need to solve?  $y$

E. Solve for the remaining variable. Then write the solution as a point in  $(x, y, z)$  form.

$$\begin{array}{l} \textcircled{1} \quad 2\left(\frac{1}{2}\right) + 3y + 3(2) = 6 \\ 1 + 3y + 6 = 6 \\ 3y + 7 = 6 \\ 3y = -1 \\ y = -\frac{1}{3} \end{array}$$

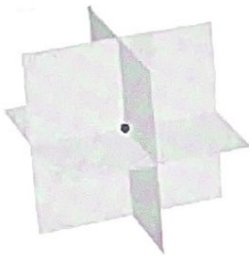
$$\boxed{\left(\frac{1}{2}, -\frac{1}{3}, 2\right)}$$

## Systems with 3 VARIABLES

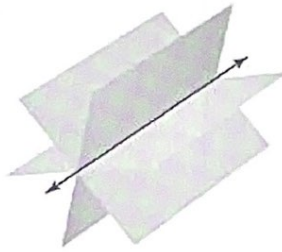
1. Choose 2 equations \_\_\_\_\_ in which you can easily eliminate a variable. Let's call this new equation Equation A.
2. Eliminate the same variable from the unused equation and any other equation. Let's call this new equation Equation B.
3. Use Equation A + B \_\_\_\_\_ and SOLVE that system.
4. Substitute your answer from Step 3 into any of the original equations find the remaining variable. Your solution is  $(x,y,z)$ .

coordinate triplet

## Types of Solutions to a System with 3 Variables



(a) One solution  
(a point)



(b) Infinite number  
of solutions (a line)



(c) Infinite number  
of solutions (a plane)



(d) No solution



(e) No solution

Additional Example

$$\begin{array}{l} \textcircled{1} \quad 2x - 6y + 9z = -8 \\ \textcircled{2} \quad 5x + y + 2z = 10 \\ \textcircled{3} \quad 3x + y - 8z = -28 \end{array}$$

$$\begin{array}{r} \textcircled{2} \quad 5x + y + 2z = 10 \\ \quad -3x - y + 8z = 28 \\ \hline \end{array} \quad \begin{array}{l} \leftarrow x(-1) \end{array}$$

$$\textcircled{A} \quad 2x + 10z = 38$$

$$\begin{array}{r} \textcircled{1} \quad 2x - 6y + 9z = -8 \\ \quad 30x + 6y + 12z = 60 \\ \hline \end{array} \quad \begin{array}{l} \leftarrow x(6) \end{array}$$

$$\textcircled{B} \quad 32x + 21z = 52$$

$$\textcircled{A} \quad -32x - 160z = -608$$

$x(-16)$

$$\begin{array}{r} \textcircled{B} \quad 32x + 21z = 52 \\ \hline \quad -139z = -556 \end{array}$$

$$\textcircled{Z} = 4$$

$$\begin{array}{l} \textcircled{A} \quad 2x + 10(4) = 38 \\ \quad 2x + 40 = 38 \end{array}$$

$$2x = -2$$

$$\textcircled{X} = -1$$

$$\begin{array}{l} \textcircled{3} \quad 3(-1) + y - 8(4) = -28 \\ \quad -5 + y - 32 = -28 \end{array}$$

$$y - 35 = -28$$

$$\textcircled{Y} = 7$$

solution

$$\boxed{(-1, 7, 4)}$$